Common Graphs with Arbitrary Chromatic Number

Jan Volec

(joint work with Dan Král' and Fan Wei)

Ramsey's theorem states that for every graph H, there is an integer $R(H)$ such every 2-edge-coloring of $R(H)$-vertex complete graph contains a monochromatic copy of H. In this talk, we focus on a natural quantitative extension: how many monochromatic copies of H can we find in every 2-edge-coloring of K_n, and what graphs H are so-called common, i.e., the number of monochromatic copies of H is asymptotically minimized by a random 2-edge-coloring. A classical result of Goodman [1] from 1959 states that the triangle is a common graph. On the other hand, Thomason [4] proved in 1989 that no clique of order at least four is common, and the existence of a common graph with chromatic number larger than three was open until 2012, when Hatami, Hladký, Král', Norin and Razborov [2] proved that the 5-wheel is common. In this talk, we show that for every $k > 4$, there exists a common graph with chromatic number k.

References

[1] A. W. Goodman: *On sets of acquaintances and strangers at any party*, Amer. Math. Monthly **66** (1959), 778–783.

[2] H. Hatami, J. Hladký, D. Král', S. Norine and A. Razborov: *Non-three-colourable common graphs exist*, Combin. Probab. Comput. **21** (2012), 734–742.

[3] D. Král', J. Volec, F. Wei: Common graphs with arbitrary chromatic number, Available as arxiv:2206.05800.

[4] A. Thomason: *A disproof of a conjecture of Erdős in Ramsey theory*, J. London Math. Soc. (2) **39** (1989), 246–255.