# 27th Workshop Cycles and Colourings 

Nový Smokovec, September 2-7, 2018


High Tatras, Slovakia

# Book of Abstracts 

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# Abstracts of the $\mathbf{2 7}^{\text {th }}$ Workshop Cycles and Colourings 

September 2-7, 2018, Nový Smokovec, High Tatras, Slovakia
Editors: Igor Fabrici, František Kardoš

Dear Participant,
welcome to the Twenty-seventh Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining twenty five workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994-2003, Tatranská Štrba 2004-2010, Nový Smokovec 2011-2017).
The series of C\&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks, the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

This workshop is dedicated to the 70th birthday of Stano Jendrol', who is one of the founders of the whole sequence of $\mathrm{C} \& \mathrm{C}$ workshops.

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

## Invited speakers:

Csilla Bujtás University of Pannonia, Veszprém, Hungary
Jochen Harant Technische Universität Ilmenau, Germany
Bill Jackson Queen Mary University of London, United Kingdom
Ross J. Kang Radboud University Nijmegen, Netherlands
Bojan Mohar Simon Fraser University, Burnaby, Canada
University of Ljubljana, Slovenia
Mariusz Woźniak AGH University of Science and Technology, Kraków, Poland
Carol Zamfirescu Ghent University, Belgium
Have a pleasant and successful stay in Nový Smokovec.

## Organising Committee:

Igor Fabrici<br>František Kardoš<br>Mária Maceková<br>Tomás Madaras<br>Martina Mockovčiaková<br>Roman Soták

## Programme

| Sunday |  |
| :--- | :--- |
| $16: 00-22: 00$ | Registration |
| $18: 00-21: 00$ | Dinner |


| Monday |  |  |  |
| :---: | :---: | :---: | :---: |
| 07:00-09:00 | Breakfast |  |  |
| 09:00-09:50 | A | Bujtás | Covering triangles by edges |
| 09:55-10:15 | A | Ryjáček | A closure concept for $\left\{K_{1,4}, K_{1,4}+e\right\}$-free graphs |
| 10:20-10:40 | A | Mohr | Kempe chains and rooted minors |
| 10:45-11:15 | Coffee break |  |  |
| 11:15-11:35 | A | Lužar | On 3-choosability of 4-regular planar graphs |
|  | B | Korcsok | 2-colored point-set embeddability of the outerplanar graphs |
| 11:40-12:00 | A | Pierron | Coloring the squares of planar graphs with no 4-cycle |
|  | B | Masařík | Complexity of packing coloring |
| 12:05-12:30 | A | Problem session 1 |  |
| 12:30-14:00 | Lunch |  |  |
| 15:30-16:20 | A | Harant | Lightweight paths in graphs |
| 16:25-16:55 | Coffee break |  |  |
| 16:55-17:15 | A | Czap | Zig-zag coloring of plane graphs |
|  | B | Dettlaff | Domination and certified domination numbers |
| 17:20-17:40 | A | Valiska | Facial $L(2,1)$-labelings of trees |
|  | B | Lemańska | Characterizations of some perfect graphs |
| 17:45-18:05 | A | Šugerek | Three classes of 1-planar graphs |
|  | B | Michalski | Secondary kernels in graph products |
| 19:00- | Birthday party |  |  |


| Tuesday |  |  |  |
| :---: | :---: | :---: | :---: |
| 07:00-09:00 | Breakfast |  |  |
| 09:00-09:50 | A | Jackson | Rigidity of graphs and frameworks |
| 09:55-10:15 | A | Tuza | Efficient algorithms for tropical matchings |
| 10:20-10:40 | A | Šámal | A rainbow version of Mantel's Theorem |
| 10:45-11:15 | Coffee break |  |  |
| 11:15-11:35 | A | Schmidt | Even longer cycles in essentially 4-connected planar graphs |
|  | B | Hušek | A generalization of nowhere-zero flows |
| 11:40-12:00 | A | Lo | Longest cycles in cyclically 4 -edge-connected cubic planar graphs |
|  | B | Kompišová | Flow and circular flow number of cubic signed graphs |
| 12:05-12:30 | A | Problem session 2 |  |
| 12:30-14:00 | Lunch |  |  |
| 15:30-16:20 | A | Zamfirescu | Spanning subgraphs of planar graphs |
| 16:25-16:55 | Coffee break |  |  |
| 16:55-17:15 | A | Schweser | DP-colorings of hypergraphs |
|  | B | Čekanová | Structure of edges in plane graphs with bounded dual edge weight |
| 17:20-17:40 | A | Mockovčiaková | Some leapfrog fullerene graphs have exponentially many Hamilton cycles |
|  | B | Knor | Trees with the maximal value of Graovac-Pisanski index |
| 17:45-18:05 | A | Timková | H-force number in distance graphs |
|  | B | Abrosimov | On the volume of a compact hyperbolic antiprism |
| 18:15-20:00 | Dinner |  |  |


| Wednesday |  |
| :--- | :--- |
| $06: 30-09: 00$ | Breakfast |
| $08: 00-15: 00$ | Trip |
| $13: 00-15: 00$ | Lunch |
| $18: 15-20: 00$ | Dinner |


| Thursday |  |  |  |
| :---: | :---: | :---: | :---: |
| 07:00-09:00 | Breakfast |  |  |
| 09:00-09:50 | A | Mohar | Fifty years of the Ringel and Youngs Map Colour Theorem |
| 09:55-10:15 | A | Nedela | Hamiltonicity of cubic Cayley graphs of small girth |
| 10:20-10:40 | A | Škoviera | Cyclic connectivity, edge-elimination, and the twisted Isaacs graphs |
| 10:45-11:15 | Coffee break |  |  |
| 11:15-11:35 | A | Lukot'ka | Short cycle covers of graphs with minimal degree three |
|  | B | Pekárek | Triangle-free 3-colorability on torus and cylinder |
| 11:40-12:00 | A | Máčajová | Smallest nontrivial snarks of oddness 4 |
|  | B | Goodall | A Tutte polynomial for maps |
| 12:05-12:25 | A | Mazák | Structure of small snarks |
|  | B | Steiner | Circular colourings of digraphs |
| 12:30-14:00 | Lunch |  |  |
| 15:30-16:20 | A | Woźniak | Local irregularity - a new point of view |
| 16:25-16:55 | Coffee break |  |  |
| 16:55-17:15 | A | Przybyło | Regular graphs can be decomposed into two subgraphs fulfilling the 1-2-3 Conjecture |
|  | B | Furmańczyk | Equitable list vertex colourability and arboricity of grids |
| 17:20-17:40 | A | Pelayo | Neighbor-locating colorings in pseudotrees |
|  | B | Bednarz | On new generalization of the Fibonacci numbers |
| 17:45-18:05 | A | Feñovčíková | On inclusive distance vertex irregular labelings |
|  | B | Görlich | $\mathbb{Z}_{2} \times \mathbb{Z}_{2}$-cordial cycle-free hypergraphs |
| 19:00 - | Farewell party |  |  |


| Friday |  |  |  |
| :--- | :--- | :--- | :--- |
| $07: 00-09: 00$ | Breakfast |  |  |
| 09:00-09:50 | A | Kang | $\begin{array}{l}\text { Three problems about cycles and colourings } \\ \text { 09:55-10:15 }\end{array}$ |
| A | Practional chromatic number of small degree graphs of |  |  |
| given girth |  |  |  |
| List 3-Coloring is polynomial on graphs without linear |  |  |  |
| forests up to seven vertices |  |  |  |$]$

# On the volume of a compact hyperbolic antiprism 

Nikolay Abrosimov<br>(joint work with Vuong Huu Bao)

We consider a compact hyperbolic antiprism. It is a convex polyhedron with $2 n$ vertices in $\mathbb{H}^{3}$ which has a symmetry group $S_{2 n}$ generated by a mirror-rotational symmetry of order $2 n$, i.e. rotation to the angle $\pi / n$ followed by a reflection. We establish necessary and sufficient conditions for the existence of such polyhedra in hyperbolic space $\mathbb{H}^{3}$. Then we find relations between their dihedral angles and edge lengths in the form of a cosine rule. Finally, we obtain exact integral formulas expressing the volume of a hyperbolic antiprism in terms of the edge lengths.

Theorem 1. A compact hyperbolic antiprism with $2 n$ vertices and edge lengths $a, c$ having a symmetry group $S_{2 n}$ is exist if and only if

$$
1+\cosh a-2 \cosh c+2(1-\cosh c) \cos \frac{\pi}{n}<0
$$

Theorem 2. The volume of a compact hyperbolic antiprism with $2 n$ vertices and edge lengths $a, c$ is given by the formula

$$
V=n \int_{c_{0}}^{c} \frac{a G+t H}{\left(2 \cosh ^{2} t-1-\cosh a\right) \sqrt{R}} d t,
$$

where
$G=2\left(\cosh t-\cos \frac{\pi}{n}\right) \sinh a \sinh t$,
$H=-(\cosh a-1)\left(1+\cosh a+2 \cosh ^{2} t-4 \cosh t \cos \frac{\pi}{n}\right)$,
$R=2-\cosh a(2+\cosh a)+\cosh 2 t+4(\cosh a-1) \cosh t \cos \frac{\pi}{n}-2 \sinh ^{2} t \cos \frac{2 \pi}{n}$
and $c_{0}$ is the root of the equation $2 \cosh c\left(1+\cos \frac{\pi}{n}\right)=1+\cosh a+2 \cos \frac{\pi}{n}$.
In particular case $n=3$ an antiprism become an octahedron with $\overline{3}$-symmetry. In this case theorems 1 and 2 are coincide with the results given in [1]. When $n=2$ the upper and lower $n$-gonal faces of an antiprism degenerate to line segments. Thus we get a tetrahedron with a symmetry group $S_{4}$. The latter case was previously studied in [2].

## References

[1] N.V. Abrosimov, E.S. Kudina, A.D. Mednykh, On the volume of a hyperbolic octahedron with $\overline{3}$-symmetry, Proc. Steklov Inst. Math. 288 (2015), 1-9.
[2] N.V. Abrosimov, V.H. Bao, The volume of a hyperbolic tetrahedron with symmetry group $S_{4}$, Tr. Inst. Mat. Mekh. 23:4 (2017), 7-17.

## On new generalization of the Fibonacci numbers

Natalia Bednarz

Let $k \geqslant 2, n \geqslant 0$ be integers and let $p \geqslant 1$ be a rational number. The $(k, p)$ Fibonacci numbers $F_{k, p}(n)$ are defined recursively in the following way

$$
F_{k, p}(n)=p F_{k, p}(n-1)+(p-1) F_{k, p}(n-k+1)+F_{k, p}(n-k)
$$

for $n \geqslant k$ with initial conditions

$$
F_{k, p}(n)=\left\{\begin{array}{cl}
0 & \text { for } n=0 \\
p^{n-1} & \text { for } 0<n \leqslant k-1
\end{array}\right.
$$

Particular cases of the previous definition are:

- If $k=2, p=1$, the classical Fibonacci numbers are obtained.
- If $k=2, p=\frac{3}{2}$, the Pell sequence appears.

In the talk we give some properties of numbers $F_{k, p}(n)$ and their combinatorial interpretations. In particular these interpretations are related to tilings and special edge-shade colouring in graphs. We present identities for numbers $F_{k, p}(n)$ which generalize the well-known identities for Fibonacci numbers and Pell numbers, simultaneously.

## References

[1] U. Bednarz, I. Włoch, M. Wołowiec-Musiał, Total graph interpretation of the numbers of the Fibonacci type, J. Appl. Math. (2015), 1-7.
[2] N. Bednarz, A. Włoch, I. Włoch, The Fibonacci numbers in edge coloured unicyclic graphs, Util. Math. 106 (2018), 39-49.

## Covering triangles by edges

## Csilla Bujtás

In a graph G, a triangle packing is a set of pairwise edge-disjoint triangles, and a triangle covering is a set of edges the removal of which makes the graph trianglefree. The maximum size $\nu_{\Delta}(G)$ of a triangle packing and the minimum size $\tau_{\Delta}(G)$ of a triangle covering clearly satisfies $\tau_{\Delta}(G) \leq 3 \nu_{\Delta}(G)$. It was conjectured by Zsolt Tuza in 1984 that the following stronger statement

$$
\tau_{\Delta}(G) \leq 2 \nu_{\Delta}(G)
$$

is also valid for every $G$. For the complete graphs $K_{4}$ and $K_{5}$, the relation holds with equality as $\tau_{\Delta}\left(K_{4}\right)=2, \nu_{\Delta}\left(K_{4}\right)=1$, and $\tau_{\Delta}\left(K_{5}\right)=4, \nu_{\Delta}\left(K_{5}\right)=2$.

Moreover, for every positive $\epsilon$ there exists a $K_{4}$-free graph $G$ with $\tau_{\Delta}(G)>$ $(2-\epsilon) \nu_{\Delta}(G)$.

Although the problem was extensively studied by lots of authors, the conjecture is still open. In the talk, we survey the earlier results and discuss some recent ones concentrating on the class of $K_{4}$-free graphs.

# Structure of edges in plane graphs with bounded dual edge weight 

Katarína Čekanová<br>(joint work with Mária Maceková)

In 1955 Kotzig proved that every 3-connected plane graph contains an edge with the sum of degrees of its end vertices at most 13. This result was later extended to all planar graphs having $\delta(G) \geq 3$. The plane graph $K_{2, r}, r \geq 2$, shows that analogue of Kotzig theorem cannot be extended in general for graphs with minimum degree 2. However, if we consider additional condition on the girth of the graph, then $G$ will contain an edge of weight at most 7 for $g(G) \geq 5$.

Authors Hudák, Maceková, Madaras and Široczki studied the relationship between the minimum vertex degree, minimum face size, minimum edge weight and minimum dual edge weight in plane graphs with $\delta(G)=2$. They determined for which parameters the corresponding families are nonempty or empty, respectively. This inspired us to study the structure of edges in connected plane graphs with $\delta(G)=2$ and given dual edge weight $w^{*}(G)$, where

$$
w^{*}(G)=\min \{d(\alpha)+d(\beta): \alpha, \beta \in F, \alpha \neq \beta, \alpha \text { and } \beta \text { have a common edge }\} .
$$

We proved the following: if $w^{*}(G) \geq 9$, then $G$ contains an edge of type $(2,10)$ or $(3,4)$; if $w^{*}(G) \geq 10$, then $G$ contains an edge of type $(2,10)$ or $(3,3)$; if $w^{*}(G) \geq 11$, then $G$ contains an edge of type $(2,6)$ or $(3,3)$; if $w^{*}(G) \geq 14$, then $G$ contains an edge of type $(2,6)$, and if $w^{*}(G) \geq 15$, then $G$ contains an edge of type (2, 4). Moreover, all bounds are the best possible.

## Zig-zag coloring of plane graphs

Július Czap<br>(joint work with Stanislav Jendrol' and Margit Voigt)

Let $G$ be a plane graph with vertex set $V$, edge set $E$ and face set $F$. Two distinct edges are facially adjacent in $G$ if they are consecutive edges on the boundary walk of a face of $G$. Two distinct elements of $V \cup E$ are facially adjacent in $G$ if they are incident elements, adjacent vertices or facially adjacent edges. A facial total-coloring of $G$ is a total-coloring such that any two facially adjacent elements receive different colors. A zig-zag coloring of $G$ is a facial
total-coloring $c: V \cup E \rightarrow\{1, \ldots, k\}$ such that $c\left(x_{i}\right)>\max \left\{c\left(x_{i-1}\right), c\left(x_{i+1}\right)\right\}$ or $c\left(x_{i}\right)<\min \left\{c\left(x_{i-1}\right), c\left(x_{i+1}\right)\right\}$ for every $x_{i-1} x_{i} x_{i+1} \subseteq \partial(f)$, where $\partial(f)$ denotes the boundary walk of a face $f$.

In the talk we obtain lower and upper bounds for the minimum number of colors which is necessary for such a coloring. Moreover, we give several sharpness examples and formulate some open problems.

## Domination and certified domination numbers

Magda Dettlaff<br>(joint work with Magdalena Lemańska, Mateusz Miotk, Jerzy Topp, Radosław Ziemann, and Paweł Żyliński)

Given a graph $G$, we say that a subset $D \subseteq V_{G}$ is a dominating set of $G$ if every vertex belonging to $V_{G}-D$ is adjacent to at least one vertex in $D$. The domination number (upper domination number, respectively) of a graph $G$, denoted by $\gamma(G)(\Gamma(G)$, respectively), is the cardinality of a smallest (largest minimal, respectively) dominating set of $G$. A subset $D \subseteq V_{G}$ is called a certified dominating set of $G$ if $D$ is a dominating set of $G$ and every vertex belonging to $D$ has either zero or at least twoneighbors in $V_{G}-D$. The cardinality of a smallest (largest minimal, respectively) certified dominating set of $G$ is called the certified ( upper certified, respectively) domination number of $G$ and is denoted by $\gamma_{\mathrm{cer}}(G)$ ( $\Gamma_{\text {cer }}(G)$, respectively).

It is obvious that for any graph $G$ we have the inequalities $\gamma(G) \leq \gamma_{\text {cer }}(G) \leq$ $\Gamma_{\text {cer }}(G)$, while the parameters $\gamma_{\text {cer }}(G)$ and $\Gamma(G)$, and also the parameters $\Gamma_{\text {cer }}(G)$ and $\Gamma(G)$ are not comparable.

Certified domination was introduced in [1] in order to describe some possible relations in social networks. The studies on this topic are continued in [2]. For different classes of graphs $G$ we establish conditions for the equality of the domination number $\gamma(G)$ and the certified domination number $\gamma_{\text {cer }}(G)$ of a graph $G$. Furthermore, we characterize all graphs $G$ for which $\gamma(H)=\gamma_{\text {cer }}(H)$ for each induced and connected subgraph $H \neq K_{2}$ of $G$. We also study the main properties of the upper certified domination number $\Gamma_{\text {cer }}(G)$ of $G$ and its relations to $\gamma_{\mathrm{cer}}(G)$ and $\Gamma(G)$.

## References

[1] M. Dettlaff, M. Lemańska, J. Topp, R. Ziemann, P. Żyliński, Certified domination, submitted.
[2] M. Dettlaff, M. Lemańska, M. Miotk, J. Topp, R. Ziemann, P. Żyliński, Graphs with equal domination and certified domination numbers, submitted.

# On inclusive distance vertex irregular labelings 

Andrea Feňovčíková<br>(joint work with Martin Bača, Slamin, and Kiki A. Sugeng)

For a simple graph $G$, a vertex labeling $f: V(G) \rightarrow\{1,2, \ldots, k\}$ is called a $k$-labeling. The weight of a vertex $v$, denoted by $w t_{f}(v)$ is the sum of all vertex labels of vertices in the closed neighborhood of the vertex $v$. A vertex $k$-labeling is defined to be an inclusive distance vertex irregular $k$-labeling of $G$ if for every two different vertices $u$ and $v$ there is $w t_{f}(u) \neq w t_{f}(v)$. The minimum $k$ for which the graph $G$ has an inclusive distance vertex irregular $k$-labeling is called the inclusive distance vertex irregularity strength of $G$.

In the talk we will establish some bounds of the inclusive distance vertex irregularity strength and determine the exact value of this parameter for several families of graphs.

# Equitable list vertex colourability and arboricity of grids 

Hanna Furmańczyk<br>(joint work with Ewa Drgas-Burchardt, Janusz Dybizbański, and Elżbieta Sidorowicz)

A graph $G$ is equitably $k$-list arborable if for any $k$-uniform list assignment $L$, there is an equitable $L$-colouring of $G$ whose each colour class induces an acyclic graph. The smallest number $k$ admitting such a coloring is named equitable list vertex arboricity and is denoted by $\rho_{l}^{=}(G)$. Zhang in 2016 [1] posed the conjecture that if $k \geq\lceil(\Delta(G)+1) / 2\rceil$ then $G$ is equitably $k$-list arborable. We give some new tools that are helpful in determining values of $k$ for which a general graph is equitably $k$-list arborable. We use them to prove the Zhang's conjecture [1] for $d$-dimensional grids where $d \in\{2,3,4\}$ and give new bounds on $\rho_{l}^{=}(G)$ for general graphs and for $d$-dimensional grids with $d \geq 5$.

## References

[1] X. Zhang, Equitable list point arboricity of graphs, Filomat 30:2 (2016), 373378.

# Triangle-free graphs with every matching in a Hamiltonian cycle 

## Grzegorz Gancarzewicz

We consider only finite graphs without loops and multiple edges. We will give a sufficient condition on the degrees of the vertices in triangle-free graphs under which every matching is contained in a Hamiltonian cycle. This is a modification of a result by Kenneth A. Berman [1] for cycles through matchings in graphs and a result by Stephan Brand [2] for Hamiltonian cycles in triangle-free graphs.

## References

[1] K.A. Berman, Proof of a conjecture of Haggkvist on cycles and independent edges, Discrete Math. 46 (1983) 9-13.
[2] S. Brandt, Cycles and paths in triangle-free graphs, in: R.L. Graham, J. Nešetřil (eds.) The Mathematics of Paul Erdős II, Algorithms and Combinatorics, Springer, 1997.

## $\mathbb{Z}_{2} \times \mathbb{Z}_{2}$-cordial cycle-free hypergraphs

Agnieszka Görlich<br>(joint work with Sylwia Cichacz and Zsolt Tuza)

If $A$ is an Abelian group, then a labeling $f: V(G) \rightarrow A$ of the vertices of some graph $G$ induces an edge labeling on $G$; the edge $u v$ receives the label $f(u)+f(v)$. A graph $G$ is $A$-cordial if there is a vertex-labeling such that the vertex label classes differ in size by at most one and that the induced edge label classes differ in size by at most one.

The problem of $A$-cordial labelings of graphs can be naturally extended for hypergraphs. We show some families of cycle-free $\mathbb{Z}_{2} \times \mathbb{Z}_{2}$-cordial hypergraphs i.a. $p$-uniform hypetrees for $p>2$, stars and hyperpaths. We also present a necessary and sufficient condition for hypergraphs of maximum degree 1 to be $\mathbb{Z}_{2} \times \mathbb{Z}_{2^{-}}$ cordial.

## A Tutte polynomial for maps

## Andrew Goodall

(joint work with Guus Regts, Lluis Vena, Thomas Krajewski, and Bart Litjens)
We follow Tutte's footsteps in how he created his "dichromate" (the Tutte polynomial) as a simultaneous generalization of the chromatic polynomial and flow polynomial, counting colourings and flows of graphs. Only this time we shall count the analogues of colourings and flows for maps (graphs 2-cell embedded in
surfaces). We end up with a multivariate polynomial - a "dichromate for maps" - that, like the Tutte polynomial of a graph, contains combinatorially significant specializations other than the colourings and flows it was defined to capture at the outset. Formally akin to Tutte's $V$-function for graphs, our invariant contains among its specializations the various existing extensions of the Tutte polynomial to maps, namely the Bollobás-Riordan polynomial, Las Vergnas polynomial and Krushkal polynomial.

This talk will be a broad-brush sketch of ideas contained in [1, 2].

## References

[1] A.J. Goodall, T. Krajewski, G. Regts, L. Vena, A Tutte polynomial for maps, Combin. Probab. Comput., to appear, arXiV:1610.04486.
[2] A.J. Goodall, B.M. Litjens, G. Regts, L. Vena, A Tutte polynomial for maps II: the non-orientable case, arXiv:1804.01496.

## Lightweight paths in graphs

## Jochen Harant <br> (joint work with Meret Behrens and Stanislav Jendrol')

For a path $P$ on $V(P)$ of a graph $G$, let $w_{G}(P)=\sum_{v \in V(P)} d_{G}(v)$ be the weight of $P$ in $G$, where $d_{G}(v)$ denotes the degree of a vertex $v$ in $G$. Clearly, if $P$ is a hamiltonian path of $G$ and $d$ is the average degree of $G$, then $w_{G}(P)=d \cdot|V(G)|$.

We investigate the following problem:
Given a graph $G$ on $V(G)$ and an integer $l<|V(G)|$, find the smallest constant $c=c(G, l)$ such that whenever $G$ contains a path on $k \leq l$ vertices, there is a path $P$ of $G$ on $k$ vertices for which $w_{G}(P) \leq c \cdot k$.

## A generalization of nowhere-zero flows

Radek Hušek<br>(joint work with Peter Korcsok and Robert Šámal)

A flow in a digraph $G=(V, E)$ is an assignment of values of some abelian group $\Gamma$ to edges of $G$ such that Kirchhoff's law is valid at every vertex. We say a flow is nowhere-zero if it does not use value 0 at any edge.

A natural generalization of nowhere-zero flows are $k$-free flows (first introduced as $k$-connected flows in diploma thesis of Šámal [1]). A $\Gamma$-flow $\varphi$ is $k$-free if for $k^{\prime} \leq k$ and edges $e_{1}, \ldots, e_{k^{\prime}}$ :

$$
\varphi\left(e_{1}\right)+\cdots+\varphi\left(e_{k^{\prime}}\right) \neq 0
$$

Obviously nowhere-zero flows are exactly 1-free flows and antisymmetric flows studied by Nešetřil and Raspaud [2] and others are 2-free flows. For both nowherezero flows and antisymmetric flows there exist finite groups such that every graph has a nowhere-zero (resp. antisymmetric) flow in the given group if the graph does not contain any obvious obstacle.

For nowhere zero flows such a group is $\mathbb{Z}_{6}$ as proved by Seymour [3] and the obstacle is a bridge. In the case of antisymmetric flows the obstacles are bridges and directed cuts of size two, and it was proved by DeVos et al. [4] that some group of size less than $10^{12}$ is enough. It was asked by Nešetřil and Šámal [1] and then again in 2015 at CanaDAM conference whether for every $k$ exists some $n_{k}$ such that every graph without directed cut of size $\leq k$ (which is an obvious obstacle) does have a $k$-free flow in some group of size at most $n_{k}$.

Šámal [1] showed that this is true for $(2 k+1)$-connected graphs. We show some partial results towards this conjecture: We improve the connectivity bound to $2 k-1$ and show that it is enough to restrict ourselves to cyclic groups:

Theorem. If graph has a $k$-free flow in a group $\Gamma$, it also has a $k$-free flow in $\mathbb{Z}_{n}$ for all $n \geq f(|\Gamma|, k)$.

## References

[1] R. Šámal, Nenulové toky, diploma thesis 2001.
[2] J. Nešetřil, A. Raspaud, Antisymmetric flows and strong colourings of oriented graphs, Ann. Inst. Fourier 49:3 (1999), 1037-1056.
[3] P. Seymour, Nowhere-zero 6-flows, J. Combin. Theory Ser. B 30:2 (1981), 130-135.
[4] M. DeVos, T. Johnson, P. Seymour, Cut coloring and circuit covering, manuscript 2003.

## Rigidity of graphs and frameworks

## Bill Jackson

The first reference to the rigidity of frameworks in the mathematical literature occurs in a problem posed by Euler in 1776. Consider a polyhedron $P$ in 3space. We view $P$ as a 'panel-and-hinge framework' in which the faces are 2 dimensional panels and the edges are 1-dimensional hinges. The panels are free to move continuously in 3 -space, subject to the constraints that the shapes of the panels and the adjacencies between them are preserved, and that the relative motion between pairs of adjacent panels is a rotation about their common hinge. The polyhedron $P$ is rigid if every such motion results in a polyhedron which is congruent to $P$. Euler's conjecture was that every polyhedron is rigid.

The conjecture was verified for the case when $P$ is convex by Cauchy in 1813. Gluck showed in 1975 that it is true when $P$ is 'generic' i.e. there are no algebraic dependencies between the coordinates of the vertices of $P$. Connelly finally disproved the conjecture in 1982 by constructing a polyhedron which is not rigid.

I will describe results and open problems concerning the rigidity of various other types of frameworks. I will be mostly concerned with the generic case for which the problem of characterizing rigidity usually reduces to pure graph theory.

## Three problems about cycles and colourings

Ross J. Kang

My aim is to present three recently posed problems. They touch upon several different areas of combinatorial mathematics. For each of them I will briefly give background and motivation. The first two relate well to work by Stano Jendrol'.

Conjecture 1 ([3]). Fix $t \geq 2$. There is some even $\lambda_{t}$ such that for any even $\ell$ the following holds. There are $C_{\ell}$-free graphs of maximum degree $d$ with distance- $t$ chromatic index $\Omega\left(d^{t}\right)$ if $\ell<\lambda_{t}$, while every $C_{\ell}$-free graph of maximum degree $d$ has distance- $t$ chromatic index $O\left(d^{t} / \log d\right)$ if $\ell \geq \lambda_{t}$.

Conjecture 2 ([2]). Fix $\varepsilon>0$. For $d$ sufficiently large, every planar multigraph of maximum degree $d$ has strong chromatic index at most $(9 / 2+\varepsilon) d$.

Conjecture 3 ([1]). There exists $C>0$ such that any triangle-free graph of minimum degree $d$ contains a bipartite induced subgraph of minimum degree $C \log d$.

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## Trees with the maximal value of Graovac-Pisanski index

Martin Knor<br>(joint work with Riste Škrekovski and Aleksandra Tepeh)

Let $G$ be a graph. Its Graovac-Pisanski index is defined as

$$
G P(G)=\frac{|V(G)|}{2|\operatorname{Aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \operatorname{Aut}(G)} \operatorname{dist}_{G}(u, \alpha(u)),
$$

where $\operatorname{Aut}(G)$ is the group of automorphisms of $G$. Similarly as Wiener index is correlated with boiling points of alcanes, Graovac-Pisanski index is correlated
with melting points of hydrocarbon molecules. Obviously, if the group of automorphisms of $G$ is trivial, then $G P(G)=0$. Interesting is the opposite problem. We proved that if $T$ is a tree on $n$ vertices with the maximum value of GraovacPisanski index, $n \geq 8$, then $T$ is either a path on $n$ vertices $P_{n}$, or a graph obtained from $P_{n-4}$ by attaching two pendant vertices to each end of the path.

# Flow and circular flow number of cubic signed graphs 

Anna Kompišová<br>(joint work with Edita Máčajová)

A signed graph $(G, \sigma)$ is a graph $G$ with signature $\sigma: E \rightarrow\{1,-1\}$. The flow number $\Phi(G, \sigma)$ of a flow-admissible signed graph $(G, \sigma)$ is the smallest integer $k$, for which there exists an integer nowhere-zero $k$-flow on $(G, \sigma)$. The circular flow number $\Phi_{c}(G, \sigma)$ of a flow-admissible signed graph $(G, \sigma)$ is the infimum of real numbers $r$ for which there exists an $\mathbb{R}$-flow on $(G, \sigma)$ satisfying that absolute values of all the flow values are in the interval $[1, r-1]$.

The relationship between the flow number $\Phi(G)$ and the circular flow number $\Phi_{c}(G)$ in the unsigned case is simple: $\Phi(G)=\left\lceil\Phi_{c}(G)\right\rceil$. Based on this result Raspaud and Zhu [4] conjectured, that $\Phi(G, \sigma)-\Phi_{c}(G, \sigma)<1$ for every flowadmissible signed graph $(G, \sigma)$. This conjecture was disproved by Schubert and Steffen [5], who showed that the difference can be 2. Later, Máčajová and Steffen [3] found a class of signed graphs with flow number 5 and the circular flow number converging to 2 , but the maximum degee in such graphs increases.

In this talk we concentrate on flows on cubic graphs since they play a crucial role in many open problems in graph theory. We prove that there are no signed cubic graphs with circular flow number strictly between 3 and 4 . This means, that signed cubic graph with $\Phi(G, \sigma)=3$ or 4 has the same circular flow number and if $\Phi(G, \sigma)=5$ then $\Phi_{c}(G, \sigma) \in[4,5]$. We also have found infinitely many bridgeless signed cubic graphs with $\Phi(G, \sigma)=5$ and $\Phi_{c}(G, \sigma)=4$ which disproves the conjecture of Raspaud and Zhu even for bridgeless signed cubic graphs. If we combine our result with those in $[1,2]$, we prove that for every rational number $r \in[4,5]$ there are infinitely many signed cubic graphs with $\Phi(G, \sigma)=5$ and $\Phi_{c}(G, \sigma)=r$. Finally, we prove that every known signed graph with $\Phi(G, \sigma)=6$ has also $\Phi_{c}(G, \sigma)=6$.

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# 2-colored point-set embeddability of the outerplanar graphs 

Peter Korcsok<br>(joint work with Michael Skotnica)

Given a planar graph $G=(V, E)$, we define a $k$-coloring (not necessary a proper coloring) of $G$ as a decomposition of the vertex set into $k$ disjoint sets $V=$ $V_{1} \cup V_{2} \cup \cdots \cup V_{k}$ and a color of a vertex $v \in V$ as $c(v)=i$ such that $v \in V_{i}$. For a set $S$ of points in a plane, we define a $k$-coloring as a decomposition into $k$ disjoint sets $P=P_{1} \cup P_{2} \cup \cdots \cup P_{k}$ and a color of a point $p \in P$ as $c(p)=i$ such that $p \in P_{i}$. For a $k$-colored planar graph $G$ and $k$-colored point-set $P$, we say that $G$ and $S$ are compatible if $\left|V_{i}\right|=\left|P_{i}\right|$ holds for each $i$.

Given a $k$-colored planar graph $G$ and a compatible $k$-colored point-set $P$, we want to find an embedding of $G$ into the plane such that

- each vertex $v \in V$ is embedded into a distinct point $p \in P$ such that $c(v)=c(p)$,
- each edge is embedded into partially-linear curve.

We say this embedding has a curve complexity $b$ if $b$ is the smallest integer such that each edge has at most $b$ bends.

Finally, given a family $\mathcal{G}$ of graphs, a curve complexity of $\mathcal{G}$ is the worst-case curve complexity of any $k$-colored graph $G \in \mathcal{G}$ and any compatible $k$-colored point-set $P$.

We are trying to bound the curve complexity for the 2-colored graphs. For the family of paths, it is known that both upper and lower bound is equal $1[2,4]$. Recently, Hančl [3] showed that 1 bend is enough also for the family of caterpillars. For the family of outerplanar graphs, Di Giacomo et al. [1] showed upper bound of 5 bends per edge. We show that at most 4 bends per edge are sufficient.

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# Characterizations of some perfect graphs 

Magdalena Lemańska<br>(joint work with Sergio Bermudo and Magda Dettlaff)

Given two types of graph theoretical parameters $\rho$ and $\sigma$, we say that a graph $G$ is ( $\sigma-\rho$ )-perfect if $\sigma(H)=\rho(H)$ for every non-trivial connected induced subgraph $H$ of $G$. We characterize $\left(\gamma_{w}-\tau\right)$-perfect graphs, $\left(\gamma_{w}-\alpha^{\prime}\right)$-perfect graphs, and $\left(\alpha^{\prime}-\tau\right)$-perfect graphs, where $\gamma_{w}(G), \tau(G)$ and $\alpha^{\prime}(G)$ denote the weakly connected domination number, the vertex cover number and the matching number of $G$, respectively. Moreover, we give conditions on a graph to have equalities between these three parameters.

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## Longest cycles in cyclically 4-edge-connected cubic planar graphs

On-Hei Solomon Lo
(joint work with Jens M. Schmidt)
A graph $G$ is essentially 4-connected if it is 3-connected and, for every 3-separator $S$ of $G, G-S$ has a component that is a single vertex. The shortness coefficient $\rho(\mathcal{F})$ of an infinite graph class $\mathcal{F}$ is defined to be $\lim _{\inf }^{G \in \mathcal{F}, n \rightarrow \infty} \boldsymbol{\operatorname { c i r c } ( G )} \underset{n}{ }$, where $n$
denotes the number of vertices of $G$ and $\operatorname{circ}(G)$ the length of a longest cycle in G. Grünbaum and Malkevitch [1] proved in 1976 that the shortness coefficient of cyclically 4-edge-connected cubic planar graphs is at most $\frac{76}{77}$. We show that it is at most $\frac{359}{366}\left(<\frac{52}{53}\right)$.

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# Short cycle covers of graphs with minimal degree three 

Robert Lukot'ka<br>(joint work with Anna Kompišová)

Let $G$ be a bridgeless graph. A cycle cover of $G$ is a set of circuits containing every edge of $G$. The length of a cycle cover is the sum of lengths of its circuits. Short cycle conjecture asserts that each bridgeless graph $G$ has a cycle cover of length at most $1.4 \cdot|E(G)|$. For general graphs the best bound $5 / 3 \cdot|E(G)| \approx 1.666 \cdot|E(G)|$ $[1,2]$ was proven already in the 80 's.

In this talk we restrict ourselfs to graphs without vertices of degree two. Kaiser et al. proved that such graphs have cycle cover of length at most $\approx 1.630 \cdot|E(G)|$ using a componation of methods from [1] and [2] which is also a principal approach we rely on. These ideas were refined by Fan [4] who proved the bound $\approx 1.615 \cdot|E(G)|$. Both results were obtained for loopless graphs but can be generalized to graphs with loops [3]. In this talk we sketch how to show that every bridgeless graph without vertices of degree two has a cycle cover of length less than $1.589 \cdot|E(G)|$.

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# On 3-choosability of 4-regular planar graphs 

Borut Lužar<br>(joint work with François Dross, Mária Maceková, and Roman Soták)

The question which planar graphs are 3 -colorable is well investigated. Starting with Heawood, who showed that a plane triangulation is 3 -colorable if and only if all its vertices have even degrees, it continued by Grötzsch's result showing that every triangle-free planar graph is 3 -colorable. Allowing some triangles in a graph, but still retaining 3 -colorability yielded two intriguing conjectures. First, Havel conjectured that a 3-colorable planar graph may contain many triangles as long as they are sufficiently far apart. This conjecture was recently proved by Dvořák, Král', and Thomas [3]. The second conjecture is due to Steinberg. It allows arbitrary many triangles but it forbids short cycles. Namely, Steinberg conjectured that every planar graph without cycles of length 4 and 5 is 3 -colorable. The conjecture was disproved by Cohen-Addad et al. [1].

In our talk, we present a result showing that a 4-regular planar graph obtained as the medial graph of a bipartite plane graph is 3 -choosable. This answers a question asked by Czap, Jendrol', and Voigt [2, Problem 3.9].

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## Smallest nontrivial snarks of oddness 4

Edita Máčajová<br>(joint work with Jan Goedgebeur and Martin Škoviera)

The oddness of a bridgeless cubic graph is the smallest number of odd circuits in a 2-factor of the graph. Oddness constitutes one of the most important measures of uncolourability of cubic graphs. Small graphs of given oddness are particularly important for the verification of numerous conjectures about snarks.

In [arXiv:1712.07867] we have proved that the smallest number of vertices of a snark with cyclic connectivity 4 and oddness 4 is 44 . In this talk we show that there are exactly 31 such snarks, all of them having girth 5 . These snarks are built up from subgraphs of the Petersen graph and a small number of additional vertices. Depending of their structure they fall into six classes. We indicate the
reason why these snarks have oddness 4 and sketch the proof that the 31 snarks form a complete set snarks with cyclic connectivity 4 and oddness 4 on 44 vertices.

# Complexity of packing coloring 

## Tomáš Masařík

(joint work with Minki Kim, Bernard Lidický, and Florian Pfender)
A packing $k$-coloring for some integer $k$ of a graph $G=(V, E)$ is a mapping $\varphi: V \rightarrow\{1, \ldots, k\}$ such that any two vertices $u, v$ of color $\varphi(u)=\varphi(v)$ are in distance at least $\varphi(u)+1$. This concept is motivated by frequency assignment problems. The packing chromatic number of $G$ is the smallest $k$ such that there exists a packing $k$-coloring of $G$.

Fiala and Golovach [1] showed that determining the packing chromatic number for chordal graphs is NP-complete for diameter exactly 5. While the problem is easy to solve for diameter 2, we show NP-completeness for any diameter at least 3. Our reduction also shows that the packing chromatic number is hard to approximate within $n^{1 / 2-\varepsilon}$ for any $\varepsilon>0$.

Theorem. Packing chromatic number is NP-complete on chordal graphs of any diameter at least 3. Moreover, it is hard to approximate within $n^{1 / 2-\varepsilon}$ for any $\varepsilon>0$, unless NP $=$ ZPP.


Figure 1: The reduction from Theorem on a 4 -cycle.
In addition, we design an FPT algorithm for interval graphs of bounded diameter. This leads us to exploring the problem of finding a partial coloring that maximizes the number of colored vertices. We also present some approaches to tackle the problem on (unit) interval graphs. However, the main complexity classification remains open.

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# Structure of small snarks 

Ján Mazák<br>(joint work with Jozef Rajník and Martin Škoviera)

We analyze the structure of all critical cyclically 5 -connected snarks with up to 36 vertices and take a closer look at the most interesting specimens. Based on this analysis, we generalize certain individual snarks into infinite families and construct a rather rich infinite class of cyclically 5 -connected irreducible snarks.

Certain parts of the analysis can be extended to solve Problem 5.7 proposed by Chladný and Škoviera in [1] by demonstrating that there exists a pair of nonremovable edges in an irreducible snark which is not essential.
(We say that a pair of edges of a snark $S$ is non-removable if their removal results in a3-edge-colourable graph. A pair of distinct edges $\{e, f\}$ of a snark $G$ is essential if it is non-removable and for every 2 -valent vertex $v$ of the graph $G-\{e, f\}$, the graph obtained from $G-\{e, f\}$ by suppressing $v$ is colourable.)

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## Secondary kernels in graph products

Adrian Michalski<br>(joint work with Iwona Włoch)

A subset $J \subset V(G)$ is said to be $(1,2)$-dominating if every vertex $v \notin J$ has a neighbour in $J$ and there exists another vertex in $J$ within the distance at most two from $v$. A (1,2)-kernel (also called secondary kernel) is a subset which is both independent and (1,2)-dominating. Hedetniemi et al. in [1] claimed that the problem of existence of (1,2)-kernels in an arbitrary graph is $\mathcal{N P}$ - complete and gave a sufficient condition for a graph to have a (1,2)-kernel.

Theorem. [1] Every connected graph $G$ having at least two nonadjacent vertices and no triangles has a (1,2)-kernel of cardinality $\alpha(G)$.

In the talk we present some results concerning (1,2)-kernel parameters in graphs. Moreover we give necessary and sufficient conditions for the existence of $(1,2)$ kernels in special graph products. The counting problem is also studied with the help of numbers of the Fibonacci type.

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# Some leapfrog fullerene graphs have exponentially many Hamilton cycles 

Martina Mockovčiaková<br>(joint work with František Kardoš)

3-connected planar cubic graphs with pentagonal and hexagonal faces are called fullerene graphs. A fullerene is called a leapfrog fullerene, if it can be constructed from other fullerene graph $G$ by a leapfrog transformation - it can be obtained by truncating the dual of $G$. The class of leapfrog fullerenes of fullerene graphs with an odd number of faces was the first subclass of fullerene graphs that was proved to be hamiltonian by Marušič [1]. Recently, Kardoš [2] proved that all fullerene graphs are hamiltonian.

In this talk, we show that leapfrog fullerenes of fullerene graphs with an odd number of faces have exponentially many hamilton cycles.

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# Fifty years of the Ringel and Youngs Map Colour Theorem 

## Bojan Mohar

What is the smallest genus of a surface in which the complete graph $K_{n}$ can be embedded? This question, known as the Heawood problem [2], was resolved in 1968 by Ringel and Youngs [6], and its solution gave birth to topological graph theory.

In the 1990s, Archdeacon and Grable [1] and Rödl and Thomas [7] proved that the genus of random graphs behaves very much like the genus of complete graphs.

The speaker will outline recent results about genus embeddings of dense graphs building on the work outlined above. The work, which was originally motivated by algorithmic questions [3], uses contemporary notions of quasi-randomness and graph limits, and leads to interesting new problems in topological graph theory.

Substantial part of the talk will be based on recent joint work with Yifan Jing $[4,5]$.

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## Kempe chains and rooted minors

Samuel Mohr<br>(joint work with Matthias Kriesell)

A transversal of a partition is a set which contains exactly one member from each member of the partition and nothing else. We study the following problem. Given a transversal $T$ of a proper colouring $C$ of order $k$ of some graph $G$, is there a partition $H$ of a subset of $V(G)$ into connected sets such that $T$ is a transversal of $H$ and such that two sets of $H$ are adjacent if their corresponding vertices from $T$ are connected by a path using only two colours?

This is open for each $k \geq 5$; here we consider some positive results if $k=5$, in the case that $G$ is a line graph, and disprove it for $k=7$.

# Hamiltonicity of cubic Cayley graphs of small girth 

Roman Nedela<br>(joint work with Elham Aboomahigir)

In 1969 Lovász conjectured that a vertex transitive graph admits a hamilton path. In fact, only 5 non-hamiltonian vertex transitive graphs are known, namely $K_{2}$, the Petersen and the Coxeter graphs and their truncations. This motivates a folklore conjecture stating that every Cayley graph is hamiltonian. Moreover, four of the five examples are cubic graphs.

In this talk we investigate the conjecture for the cubic Cayley graphs of girth at most 6 . In general, no non-hamiltonian cubic cyclically 7 -connected graph except
the Coxeter graph is known. Note that the cyclic connectivity of a cubic graph is bounded by the girth, and it was proved in 1995 by Nedela and Škoviera that for the vertex transitive cubic graphs the girth equals the cyclic connectivity. The fact that all known non-hamiltonian cubic graphs have cyclic connectivity bounded by 7 probably motivated Thomassen to formulate the following conjecture: If the cyclic connectivity of a cubic graph $X$ is large enough, then $X$ is hamiltonian. Even the following strong conjecture could hold: A cyclically 7-connected cubic graph is hamiltonian, or it is the Coxeter graph. Assuming the conjecture holds true, to confirm the folklore conjecture for cubic Cayley graphs it is sufficient to verify it for cubic Cayley graphs of girth $\leq 6$.

We characterize cubic Cayley graphs of girth at most six and identify few "hard families" of cubic Cayley graphs of small girth for which we are not able to verify the hamiltonicity.

# List 3-Coloring is polynomial on graphs without linear forests up to seven vertices 

Jana Novotná<br>(joint work with Tereza Klimošová, Josef Malík, Tomáš Masařík, Danïel Paulusma, and Veronika Slívová)

The $k$-Colouring problem is to decide if the vertices of a graph can be coloured with at most $k$ colours for a fixed integer $k$ such that no two adjacent vertices are coloured alike. If each vertex $u$ must be assigned a colour from a prescribed list $L(u) \subseteq\{1, \ldots, k\}$, then we obtain the List $k$-Colouring problem. A graph $G$ is $H$-free if $G$ does not contain $H$ as an induced subgraph. We continue an extensive study into the complexity of these two problems for $H$-free graphs. The graph $P_{r}+P_{s}$ is the disjoint union of the $r$-vertex path $P_{r}$ and the $s$-vertex path $P_{s}$.

We prove that List 3-Colouring is polynomial-time solvable for $\left(P_{2}+P_{5}\right)$ free graphs and for $\left(P_{3}+P_{4}\right)$-free graphs. Combining our results with known results yields complete complexity classifications of 3-Colouring and List 3Colouring on $H$-free graphs for all graphs $H$ up to seven vertices. We also prove that 5-Colouring is NP-complete for $\left(P_{3}+P_{5}\right)$-free graphs.

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# Triangle-free 3-colorability on torus and cylinder 

Jakub Pekárek<br>(joint work with Zdeněk Dvořák)

For any triangle-free graph $G$ embedded in any surface $\Sigma$ and $k \leq 4$, the $k$ colorability can be characterized by a finite list of $k$-critical subgraph obstructions (dependent on $\Sigma$ and $k$ ). We focus on triangle-free 3-colorability as the only (non-trivial) triangle-free case where the number of problematic structures is infinite. In this setting, the torus is the simplest surface with no previously known characterization of triangle-free 3-colorability.

Based on our previous results [1], characterizing all possible triangle-free 4-critical graphs embeddable in the torus, we study the problems of deciding 3-colorability and finding a proper 3 -coloring of a triangle-free graph embedded in the torus as well as the naturally related problems of triangle-free 3-colorability of graphs embedded in the cylinder with precolored faces. We present efficient algorithms for these problems.

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# Neighbor-locating colorings in pseudotrees 

Ignacio M. Pelayo<br>(joint work with Liliana Alcon, Marisa Gutierrez, Carmen Hernando, and Merce Mora)

A (proper) coloring of a graph $G$, i.e., a partition $\Pi=\left\{S_{1}, \ldots, S_{k}\right\}$ of $V(G)$ into independent sets (called colors), is said to be neighbor-locating (an NL-coloring for short) if for every pair of vertices $u, v$ belonging to color $S_{i}$, there is a color $S_{j}$ such that either $N(u) \cap S_{j} \neq \emptyset$ and $N(v) \cap S_{j}=\emptyset$ or $N(u) \cap S_{j}=\emptyset$ and $N(v) \cap S_{j} \neq \emptyset$.

The neighbor-locating chromatic number $\chi_{N L}(G)$, the $N L C$-number for short, is the minimum cardinality of an NL-coloring of $G$ [1].

Given a $k$-coloring $\Pi=\left\{S_{1}, \ldots, S_{k}\right\}$ of $V(G)$ and a vertex $x \in V(G)$, the tuple $n r(x \mid \Pi)=\left(x_{1}, \ldots, x_{k}\right)$ is a defined as follows:

$$
x_{i}= \begin{cases}0, & \text { if } x \in S_{i} \\ 1, & \text { if } x \in N\left(S_{i}\right) \backslash S_{i} \\ 2, & \text { if } x \notin N\left[S_{i}\right]\end{cases}
$$

With this terminology, $\Pi$ is an NL-coloring if and only if $n r(x \mid \Pi) \neq n r(y \mid \Pi)$, for every pair of distinct vertices $x$ and $y$.
Let $G$ be a connected graph of order $n$ with $\chi_{N L}(G)=k$. Then, $n \leq k\left(2^{k-1}-1\right)$. Moreover, if $G$ is a pseudotree (resp. a tree) then, $n \leq \frac{1}{2} k(k-1)(k+2)$ (resp. $\left.n \leq \frac{1}{2} k(k-1)(k+2)-2\right)$. In all cases, these bounds are tight, whenever $k \geq 6$.
For every integer $k \geq 4$, let $\ell(k)=k \cdot\binom{k}{2}$. If $\ell(k-1)<n \leq \ell(k)$, then

- $\chi_{N L}\left(P_{n}\right)=k$,
- $\chi_{N L}\left(C_{n}\right)=k$, if $n \neq \ell(k)-1$,
- $\chi_{N L}\left(C_{n}\right)=k+1$, if $n=\ell(k)-1$.


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# Coloring the squares of planar graphs with no 4-cycle 

## Théo Pierron

(joint work with Ilkyoo Choi and Daniel W. Cranston)
Coloring the square of a graph consists in assigning colors to its vertices in such a way that any two vertices at distance at most 2 receive different colors. While $\Delta+1$ colors are needed to properly color the square of any graph of maximum degree $\Delta$, Wegner proved that there is no hope for a matching upper bound of $\Delta+O(1)$ in general, even for planar graphs.

We study the cycle obstructions needed to obtain such a bound for planar graphs. We present here the case where only finitely many cycles lengths are forbidden. In this setting, we prove that removing only cycles of length 4 is necessary and sufficient to obtain the desired bound. For very large $\Delta$, we also improve a result of Bonamy, Cranston and Postle by showing that $\Delta+2$ colors are always sufficient for coloring the square of $C_{4}$-free planar graphs, which is tight.

# Fractional chromatic number of small degree graphs of given girth 

François Pirot<br>(joint work with Jean-Sébastien Sereni)

It is well known that you can color a graph $G$ of maximum degree $d$ greedily with $d+1$ colors. Moreover, this bound is tight, since it is reached by the cliques. Johansson proved with a pseudo-random coloring scheme that you can color triangle-free graphs of maximum degree $d$ with no more than $O(d / \log d)$ colors. This result has been recently improved to $(1+\varepsilon)(d / \log d)$ for any $\varepsilon>0$ when $d$ is big enough. This is tight up to a multiplicative constant, since you can pseudo-randomly construct a family of graphs of maximum degree $d$, arbitrary large girth, and chromatic number bigger than $d /(2 \log d)$. Although these are really nice results, they are only true for big degrees, and there remains a lot to say for small degree graphs.

When the graphs are of small degree, it is interesting to consider the fractional chromatic number instead, since it has infinitely many possible values - note that if $G$ is a subcubic graphs, then either $G=K_{4}, G$ is bipartite, or $\chi(G)=3$. It has already been settled that the maximum fractional chromatic number over the triangle-free subcubic graphs is $14 / 5$ [1]. I will present a systematic method to compute upper bounds for the independence ratio of graphs of given (small!) degree and girth, which can sometimes lead to upper bounds for the fractional chromatic number, and can be adapted to any family of small degree graphs under some local constraints.

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# Regular graphs can be decomposed into two subgraphs fulfilling the $1-2-3$ Conjecture 

Jakub Przybyło

(joint work with Julien Bensmail)
The famous 1-2-3 Conjecture asserts that the edges of every graph $G$ without a $K_{2}$-component can be weighted with 1,2 and 3 so that adjacent vertices in $G$ are associated with distinct sums of incident weights. This is still open. In the talk we show that for almost every positive integer $d$, each $d$-regular graph can be edge-decomposed into two subgraphs fulfilling the 1-2-3 Conjecture.

# Facially proper unique-maximum coloring of plane graphs 

Simona Rindošová<br>(joint work with Igor Fabrici and Mirko Horňák)

Two edges of a plane graph are facially adjacent, if they are adjacent and consecutive in a cyclic order around their end vertex. Facially proper edge (total) coloring of a plane graph is a coloring in which every two facially adjacent edges (as well as every two adjacent vertices and every edge with its end vertex) have different colors. Unique-maximum coloring of a plane graph is a coloring in which for each face the maximum color occurs exactly once on its elements (vertex or edge).
In this talk we deal with facially proper unique-maximum edge (total) coloring of plane graphs and their list versions and we present upper bounds on the corresponding chromatic numbers.

## A closure concept for $\left\{K_{1,4}, K_{1,4}+e\right\}$-free graphs <br> Zdeněk Ryjáček <br> (joint work with Petr Vrána and Shipeng Wang)

We introduce a closure concept for hamiltonicity in the class of $\left\{K_{1,4}\right.$, $\left.K_{1,4}+e\right\}$-free graphs, extending the closure for claw-free graphs. The closure of a $\left\{K_{1,4}, K_{1,4}+e\right\}$-free graph with minimum degree at least 6 is uniquely determined, is a line graph of a triangle-free graph, and preserves its hamiltonicity or non-hamiltonicity. As applications, we show that many results on claw-free graphs can be directly extended to the class of $\left\{K_{1,4}, K_{1,4}+e\right\}$-free graphs.

## A rainbow version of Mantel's Theorem

Robert Šámal<br>(joint work with Ron Aharoni, Matt DeVos, Sebastian Gonzales, and Amanda Montejano)

In this article we consider a colourful variant of the classical Mantel's theorem. Let $G_{1}, G_{2}, G_{3}$ be three graphs on a common vertex set $V$ and think of each graph as having edges of a distinct colour. Define a rainbow triangle to be three vertices $v_{1}, v_{2}, v_{3} \in V$ so that $v_{i} v_{i+1} \in E\left(G_{i}\right)$ (where the indices are treated modulo 3). We will be interested in determining how many edges force the existence of a rainbow triangle. Note that by taking $G_{1}=G_{2}=G_{3}$ we return to the setting of Mantel's Theorem. Throughout the paper we fix the value $\tau=\frac{4-\sqrt{7}}{9}$, so $\tau^{2} \approx 0.0226$. Our main theorem is as follows:

Theorem. Let $G_{1}, G_{2}, G_{3}$ be graphs on a common set of $n$ vertices. If $\left|E\left(G_{i}\right)\right|>$ $\frac{1+\tau^{2}}{4} n^{2}$ for $1 \leq i \leq 3$, then there exists a rainbow triangle.

This theorem is sharp in the sense that $\tau^{2}$ cannot be replaced by a smaller constant. Since the parameter $\tau$ is not rational, there does not exist a graph $G$ with $|V(G)|=n$ and $|E(G)|=\frac{1+\tau^{2}}{4} n^{2}$, and thus there is no finite tight example for our problem. However, in the setting of graph limits and graphons, this inconvenience is removed. Indeed, we can construct three growing sequences of graphs that converge to three graphons each with density $\frac{1+\tau^{2}}{2}$ and without a rainbow triangle. It seems certain that in the setting of graphons, Razborov's flag algebra machinery will give an alternate proof of our result, and be useful in extending it. (Indeed, such proof of our main theorem has been already obtained by Bernard Lidický, Florian Pfender, and Jan Volec.) Accordingly, our main goal here is to introduce a new type of question in extremal graph theory, provide a first proof that is easy to verify by hand, and to suggest some potential interesting directions to proceed. Here is one such question.

Problem. For what real numbers $\alpha_{1}, \alpha_{2}, \alpha_{3}>0$ is it true that every triple of graphs $G_{1}, G_{2}, G_{3}$ satisfying $\left|E\left(G_{i}\right)\right|>\alpha_{i} n^{2}$ must have a rainbow triangle?

# Even longer cycles in essentially 4-connected planar graphs 

Jens M. Schmidt

(joint work with Igor Fabrici, Jochen Harant, and Samuel Mohr)
A planar graph is called essentially 4-connected if it is 3 -connected and every 3separator is the neighborhood of a single vertex. We prove that every essentially 4-connected planar graph $G$ contains a cycle of length at least $\frac{5}{8}(n+2)$, where $n=|V(G)|$. This improves the previously best-known lower bound $\frac{3}{5}(n+2)$.

## DP-colorings of hypergraphs

## Thomas Schweser

In order to solve a question on list coloring of planar graphs, Dvořák and Postle [2] introduced the concept of DP-coloring, which shifts the problem of finding a coloring of a graph $G$ from a given list $L$ to finding an independent transversal in an auxiliary cover-graph $H$ with vertex set $\{(v, c) \mid v \in V(G), c \in L(v)\}$. This leads to a new graph parameter, called the DP-chromatic number $\chi_{\mathrm{DP}}(G)$ of $G$, which is an upper bound for the list-chromatic number $\chi_{\ell}(G)$ of $G$. The DPcoloring concept was anaylized in detail by Bernshteyn, Kostochka, and Pron [1] for graphs and multigraphs; they characterized DP-degree colorable multigraphs and deduced a Brooks' type result from this. In this talk, we will extend the concept of DP-colorings to hypergraphs having multiple (hyper-)edges. We characterize the DP-degree colorable hypergraphs and, furthermore, the corresponding
'bad' covers. This gives a Brooks' type result for the DP-chromatic number of a hypergraph.

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# Cyclic connectivity, edge-elimination, and the twisted Isaacs graphs 

Martin Škoviera<br>(joint work with Roman Nedela)

Edge-elimination is an operation of removing an edge together with its endvertices. We study the effect of this operation on the cyclic connectivity of a cubic graph. Disregarding a small number of cubic graphs with no more than six vertices, this operation cannot decrease cyclic connectivity by more than two. We show that apart from three exceptional graphs (the cube, the twisted cube, and the Petersen graph) every 2-connected cubic graph on at least eight vertices contains an edge whose elimination decreases cyclic connectivity by at most one. The proof reveals an unexpected behaviour of connectivity 6 , which requires a detailed structural analysis featuring the Isaacs flower snarks and their natural generalisation, twisted Isaacs graphs, as forced structures.

Our result is closely linked to several other problems concerning cubic graphs, for example, the existence of long cycles, decycling, and maximum genus embeddings of cubic graphs into orientable surfaces.

# Tetrises and Erdős-Faber-Lovász Conjecture 

Aneta Št'astná<br>(joint work with Ondřej Šplíchal)

Let's denote by $\mathcal{E F} \mathcal{L}$ the class of graphs where each graph consists of $n$ cliques of size $n$ where every two cliques share at most one vertex. Erdős-Faber-Lovász Conjecture (EFL) says that every graph $G \in \mathcal{E F} \mathcal{L}$ is $n$-colorable [1].
In tetris representation of $G \in \mathcal{E F} \mathcal{L}$, each vertex is represented by a vector $v \in\{0,1\}^{n}$ where $v_{i}=1$ means that corresponding vertex belongs to $i$-th clique (in some fixed enumeration of cliques). We denote by $\operatorname{deg}(v)$ the number of cliques where $v$ belongs. Set $T$ of vectors is a tetris if there does not exist any $1 \leq i, j \leq n, i \neq j$ such that $a_{i}=b_{i}=a_{j}=b_{j}=1$ for any two vectors $a$ and
b. Any tetris uniquely represents graph $G \in \mathcal{E F} \mathcal{F}$. We say that vectors $a, b$ are matching if $(a+b)_{i} \leq 1$ for any $1 \leq i \leq n$. Height of $T$, denoted by $h(T)$, is the minimum number of vectors which can be obtained by repeatedly replacing the matching vectors by their sums. If $T$ is tetris representing $G$ then $h(T)=\chi(G)$.

Lin and Chang [2] proved that EFL implies that class of tight bipartite graphs $\mathcal{B}_{n}$ is $n$ - or $(n-1)$-b-colorable. They proved this conjecture for the class $G_{n, k} \subset \mathcal{B}_{n}$. We obtained shorter proof of this theorem by showing that EFL holds for the class of graphs $H_{G_{n, k}} \subseteq \mathcal{E} \mathcal{F} \mathcal{L}$ of graphs corresponding to $G_{n, k}$.

We also used the tetris representation to generalize partial result for the dense graphs by Sánchez-Arroyo [3] who proved that EFL holds for all graphs where each vertex is contained either in exactly 1 clique, or in more than $\sqrt{n}$ cliques. Actually, he proved stronger invariant in his proof: For any vertex $v$, denote by $k$ the number of all vertices $u$ such that $\operatorname{deg}(u) \geq \operatorname{deg}(v)$ and $u$ and $v$ are not matching. Then $k \leq \frac{\operatorname{deg}(v)}{\operatorname{deg}(v)-1}(n-\operatorname{deg}(v))+1$. We generalized this to $k \leq \frac{\operatorname{deg}(v)}{\operatorname{deg}(v)-1-p}(n-\operatorname{deg}(v)-z+p)+1+p$ for any set of $p<\operatorname{deg}(v)-1$ vertices with $z$ cliques containing at least one of the $p$ vertices.

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# Circular colourings of digraphs 

Raphael Steiner<br>(joint work with Winfried Hochstättler)

A $k$-colouring of a digraph according to Erdős and Neumann-Lara is defined to be a decomposition of the vertex set into $k$ subsets each of them inducing an acylic subdigraph. Since its introduction in 1980, the corresponding relative of the chromatic number, the dichromatic number of a digraph has been investigated in several research papers.

In this talk, we introduce a new notion of circular colourings for digraphs. The most basic idea of this quantity, called star dichromatic number $\vec{\chi}^{*}(D)$ of a digraph $D$ is to allow a finer subdivision of digraphs with the same dichromatic number into such which are "easier" or "harder" to colour by allowing fractional values. This is related to a coherent notion for the vertex arboricity of graphs introduced in [3] and resembles the concept of the star chromatic number of graphs introduced by Vince in [2] in the framework of digraph colouring.

Another version of circular colourings of digraphs has already been introduced in a paper by Bokal et. al. [1]. After presenting basic properties of the new quantity, including range, simple classes of digraphs, general inequalities and its relation to integer counterparts as well as other concepts of fractional colouring, we compare these two notions for digraphs and point out similarities as well as differences in certain situations. As it turns out, the star dichromatic number is a lower bound for the cirular dichromatic number of Bokal et al., but the gap between the numbers may be arbitrarily close to 1 . This is e.g. due to the fact that while the circular dichromatic number may increase is value by adding sinks and sources, the star dichromatic number, as intuitively expected, remains generelly unchanged by such operations. In the case of planar digraphs, we approach the 2-colour-conjecture of Neumann-Lara stating that simple planar digraphs are 2colourable by transferring an upper bound of 2.5 for the circular vertex arborcity from [3]. We furthermore discuss examples of simple planar digraphs with circular dichromatic number arbitrarily close to 2 and point out some open problems.

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# Three classes of 1-planar graphs 

Peter Šugerek<br>(joint work with Július Czap)

A graph is called 1-planar if it can be drawn in the plane so that each of its edges is crossed by at most one other edge. In 2014, Zhang showed that the set of all 1-planar graphs can be decomposed into three classes $C_{0}, C_{1}, C_{2}$ with respect to the types of crossings. He proved that every $n$-vertex 1-planar graph of class $C_{1}$ has a $C_{1}$-drawing with at most $\frac{3}{5} n-\frac{6}{5}$ crossings. Consequently, every $n$-vertex 1-planar graph of class $C_{1}$ has at most $\frac{18}{5} n-\frac{36}{5}$ edges. In this talk we contribute a stronger result. We show that every $C_{1}$-drawing of a 1-planar graph has at most $\frac{3}{5} n-\frac{6}{5}$ crossings. Next we present a construction of $n$-vertex 1-planar graphs of class $C_{1}$ with $\frac{18}{5} n-\frac{36}{5}$ edges. Finally, we present the decomposition of 1-planar join products.

## H -force number in distance graphs

Mária Timková

The H-force number of a hamiltonian graph $G$ is the smallest number $k$ with the property that there exists a set $W \subseteq V(G),|W|=k$, such that each cycle passing through all vertices of $W$ is hamiltonian.

For a finite set $\left\{a_{1}, a_{2}, \ldots, a_{m}\right\}, 1 \leq a_{i} \leq n$ of positive integers, the circulant graph $C_{n}\left(a_{1}, a_{2}, \ldots, a_{m}\right)$ has vertex set $\{0,1, \ldots, n-1\}$ and two vertices $u$ and $v$ of $C_{n}\left(a_{1}, a_{2}, \ldots, a_{m}\right)$ are adjacent if $u-v \equiv \pm a_{i}(\bmod n)$.

For a circulant graph $C_{n}\left(a_{1}, a_{2}, \ldots, a_{m}\right)$ we establish the exact value of H -force number.

# Efficient algorithms for tropical matchings 


#### Abstract

Zsolt Tuza (joint work with Johanne Cohen, Yannis Manoussakis, and Hong Phong Pham) Let $G^{c}=(V, E)$ be a graph, with a given coloring $c$ on its vertices. As introduced in [1], a tropical matching is a matching whose vertex set contains at least one vertex from each color class occurring in $c$. The very first question in this context is whether $G^{c}$ contains any tropical matching. Assuming that it has one, it is of interest to determine tropical matchings of extremal size. We provide efficient algorithmic solutions for problems of this kind.


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Facial $L(2,1)$-labelings of trees<br>Juraj Valiska<br>(joint work with Július Czap and Stanislav Jendrol')

Let $T$ be a tree embedded in the plane. A facial path of $T$ is any path which is a consecutive part of the boundary walk of $T$. Two edges $e_{1}$ and $e_{2}$ of $T$ are facially adjacent if they are consecutive on a facial path of $T$. Two edges $e_{1}$ and $e_{3}$ are facially semi-adjacent if there is third edge $e_{2}$ which is facially adjacent with both $e_{1}$ and $e_{3}$, and the edges $e_{1}, e_{2}$ and $e_{3}$ are consecutive (in this order) on a facial path. An edge-labeling of $T$ with labels $1,2, \ldots, k$ is a facial $L(2,1)$-edge-labeling if difference between any two facially adjacent edges is at least 2 and difference between any two facially semi-adjacent edges is at least 1.

We show that any tree $T$ admits a facial $L(2,1)$-edge-labeling with labels $1,2, \ldots, 7$, where 7 is tight. If $T$ has no vertex of degree three, then it has such a labeling with $1,2, \ldots, 6$, which is tight. If $T$ has no vertex of degree two and three, then $T$ admits a facial $L(2,1)$-edge-labeling with labels $1,2, \ldots, 5$, which is also tight. Finally, we characterize all trees which admit a facial $L(2,1)$-edge-labeling with labels $1,2,3,4$.

## Local irregularity - a new point of view

## Mariusz Woźniak

Let us consider a coloring $f$ of edges of a simple graph $G=(V, E)$. Such a coloring defines for each vertex $x \in V$ the palette of colors, i.e., the multiset of colors of edges incident with $x$, denoted by $M(x)$. These palettes can be used to distinguish the vertices of the graph. There are many papers dealing with distinguishing either all or only neighboring vertices in a graph.

In the first part of my talk we shall see a brief survey of the problems regarding distinguishing colorings and, in particular, amazing relationships with the main purpose of the conference.

In the second part, we shall consider general edge coloring $f$ of $G$ and we shall distinguish only adjacent vertices. In other words, we will deal with local irregularities.

Another approach to this problem (introduced in [1]) is based on the concept of a locally irregular graph. A locally irregular graph is a graph whose adjacent vertices have distinct degrees. We say that a graph $G$ can be decomposed into $k$ locally irregular subgraphs if its edge set may be partitioned into $k$ subsets each of which induces a locally irregular subgraph in $G$. We shall characterize all connected graphs which cannot be decomposed into locally irregular subgraphs.

The authors of [1] conjectured that apart from these exceptions all other connected graphs can be decomposed into three locally irregular subgraphs.

I'll present also some new ideas and problems introduced recently in [2]. This new approach is a bridge between colorings and decompositions.

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# Spanning subgraphs of planar graphs 

Carol T. Zamfirescu

(joint work with Gunnar Brinkmann)
In the first part of the talk, which is based on joint work with Gunnar Brinkmann, we present a generalisation of Grinberg's hamiltonicity criterion and derive some consequences. In particular, we extend Zaks' version of the criterion, which encompasses results of Gehner and Shimamoto.

In the second part we discuss strengthenings of Thomassen's theorem stating that a planar graph of minimum degree at least 4 in which every vertex-deleted subgraph is hamiltonian, must itself be hamiltonian.

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