28th Workshop

Cycles and Colourings

Nový Smokovec, September 1–6, 2019

C&C
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Book of Abstracts

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Book of Abstracts of the 28$^{\text{th}}$ Workshop Cycles and Colourings
1–6 September 2019, Nový Smokovec, High Tatras, Slovakia

Editors: Igor Fabrici, František Kardoš

Dear Participant,

welcome to the Twenty-eighth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining twenty six workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010, Nový Smokovec 2011–2018).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks, the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Marthe Bonamy	LaBRI, Université de Bordeaux, France
Tomáš Kaiser	University of West Bohemia, Plzeň, Czech Republic
Borut Lužar	Faculty of Information Studies, Novo mesto, Slovenia
Atsuhiro Nakamoto	Yokohama National University, Japan
Monika Pilśniak	AGH University of Science and Technology, Kraków, Poland
Eckhard Steffen	Universität Paderborn, Germany
Qinglin Roger Yu	Thompson Rivers University, Kamloops, Canada

Have a pleasant and successful stay in Nový Smokovec.

Organising Committee:

Igor Fabrici
František Kardoš
Mária Maceková
Tomáš Madaras
Martina Mockovčiaková
Roman Soták

Programme

Sunday	
16:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	**Nakamoto**	Coloring triangulations, even triangulations and quadrangulations on surfaces
09:55 - 10:15	A	**Ryjáček**	Hamiltonian problems in line graphs of 3-hypergraphs
10:20 - 10:40	A	**Tuza**	Strong edge coloring of graph products
10:45 - 11:15		Coffee break	
11:15 - 11:35	A	**Masařík**	Packing directed circuits quarter-integrally
	B	**Bujtás**	Bipartite graphs with close domination and k-domination numbers
11:40 - 12:00	A	**Kabela**	Packing and covering directed triangles asymptotically
	B	**Lukoťka**	A 3-edge-colouring algorithm
12:05 - 12:25	A	**Problem session**	
12:30 - 14:00		Lunch	
15:30 - 16:20	A	**Yu**	Components condition and factors in graphs
16:25 - 16:55		Coffee break	
16:55 - 17:15	A	**Hatzel**	Cyclewidth: A branch decomposition for directed graphs
	B	**Čekanová**	Light edges in the class of toroidal graphs
17:20 - 17:40	A	**Wiederrecht**	Colouring non-even digraphs
	B	**Lin**	Equitable colorings on shifted toroidal grids
17:45 - 18:05	A	**Steiner**	Even dicuts and cut minors
	B	**Luk**	Tilings of the sphere by almost equilateral pentagons
19:00 -		Welcome party	

Tuesday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	**Pilśniak**	Asymmetric colourings of infinite graphs
09:55 - 10:15	A	**Kalinowski**	On 2-distinguishable graphs
10:20 - 10:40	A	**Woźniak**	On directed versions of the 1-2-3 conjecture and the 1-2 conjecture
10:45 - 11:15		Coffee break	
11:15 - 11:35	A	**Frick**	The strong path partition conjecture
	B	**Pelikánová**	Short rainbow cycles and cuts
11:40 - 12:00	A	**de Wet**	Regular locally hamiltonian graphs
	B	**Šťastná**	Aharoni conjecture with more edges of each color
12:05 - 12:30	A	**Problem and Photo session**	
12:30 - 14:00		Lunch	
16:25 - 16:55		Coffee break	
16:55 - 17:15	A	**Schweser**	On DP-coloring of digraphs
	B	**Parczyk**	The size-Ramsey number of tight 3-uniform paths
17:20 - 17:40	A	**Mohr**	On uniquely colourable graphs
	B	**Opler**	Generalized coloring of permutations
17:45 - 18:05	A	**Hušek**	Counting double covers of planar graphs
	B	**Semanišin**	Minimum shortest path cover
18:15 - 20:00		Dinner	

Wednesday		
06:30 - 09:00		Breakfast
07:15 - 17:30		Trip
13:00 - 15:00		Lunch
18:15 - 20:00		Dinner

Thursday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	**Steffen**	Measures of edge-uncolorability of cubic graphs
09:55 - 10:15	A	**Kemnitz**	On the chromatic edge stability index of graphs
10:20 - 10:40	A	**Jendrol'**	On the cyclic coloring conjecture
10:45 - 11:15		Coffee break	
11:15 - 11:35	A	**Schiermeyer**	Polynomial χ-binding functions for P_5-free graphs
	B	**Teska**	Hamiltonicity of lexicographic product
11:40 - 12:00	A	**Pekárek**	χ-boundedness for limited induced odd-cycle packing
	B	**Heuer**	Constructing a uniquely Hamiltonian infinite graph all whose vertices and ends have degree 3
12:05 - 12:25	A	**Novotná**	Harnessing the power of atoms
	B	**Chen**	Finding longest paths in graphs associated with hybrid wireless sensor networks
12:30 - 14:00		Lunch	
15:00 - 15:50	A	**Lužar**	Between proper and strong edge-colorings of subcubic graphs
15:55 - 16:25		Coffee break	
16:25 - 17:15	A	**Bonamy**	Planar graphs: one graph to rule them all
17:20 - 17:40	A	**Goodall**	The canonical Tutte polynomial for signed graphs
	B	**Abrosimov**	Volume of a compact hyperbolic tetrahedron in terms of its edge matrix
17:45 - 18:05	A	**Kardoš**	On the 4-color theorem for signed graphs
	B	**Knor**	Comparing Graovac-Pisanski index with Wiener index
19:00 -		Farewell party	

Friday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	**Kaiser**	Colouring Schrijver graphs: From combinatorics to topology and back again
09:55 - 10:15	A	**Suzuki**	Non-1-planarity of lexicographic products of graphs
10:20 - 10:40	A	**Horsley**	Induced path numbers of regular graphs
10:45 - 11:15		Coffee break	
11:15 - 11:35	A	**Feňovčíková**	On cycle-antimagic labelings
11:40 - 12:00	A	**Gancarzewicz**	One edge hamiltonian graphs
11:30 - 13:30		Lunch	

Volume of a compact hyperbolic tetrahedron in terms of its edge matrix

Nikolay Abrosimov
(joint work with Vuong Huu Bao)

A *compact hyperbolic tetrahedron* T is a convex hull of four points in the hyperbolic space $\mathbb{H}^3$. Let us denote the vertices of T by numbers $1, 2, 3$ and 4. Then denote by ℓ_{ij} the length of the edge connecting i-th and j-th vertices. We put θ_{ij} for the dihedral angle along the corresponding edge. It is well known that T is uniquely defined up to isometry either by the set of its dihedral angles or the set of its edge lengths. A *Gram matrix* $G(T)$ of tetrahedron T is defined as $G(T) = \langle -\cos\theta_{ij}\rangle_{i,j=1,2,3,4}$, we assume here that $-\cos\theta_{ii} = 1$. An *edge matrix* $E(T)$ of hyperbolic tetrahedron T is defined as $E(T) = \langle \cosh \ell_{ij}\rangle_{i,j=1,2,3,4}$, where $\ell_{ii} = 0$.

More than 100 years ago Italian mathematician G. Sforza found a formula for the volume of a compact hyperbolic tetrahedron T in terms of its Gram matrix (see [2]). The new proof of the Sforza's formula was recently given in [1].

In the present work we present an exact formula for the volume of a compact hyperbolic tetrahedron T in terms of its edge matrix.

Theorem. *Let T be a compact hyperbolic tetrahedron given by its edge matrix $E = E(T)$ and $c_{ij} = (-1)^{i+j}E_{ij}$ is ij-cofactor of E. We assume that all the edge lengths are fixed exept ℓ_{34} which is formal variable. Then the volume $V = V(T)$ is given by the formula*

$$V = -\frac{1}{2}\int_0^{\ell_{34}} \frac{c_{14}c_{33}(c_{24}c_{34} - c_{23}c_{44}) + c_{13}c_{44}(c_{23}c_{34} - c_{24}c_{33})}{c_{33}c_{44}\det E\sqrt{c_{33}c_{44} - c_{34}^2}}\, t\sinh t\, dt,$$

where cofactors c_{ij} and edge matrix determinant $\det E$ are functions in one variable ℓ_{34} denoted by t.

References

[1] N.V. Abrosimov, A.D. Mednykh, Volumes of polytopes in constant curvature spaces, Fields Inst. Commun. 70 (2014), 1–26.

[2] G. Sforza, Ricerche di estensionimetria differenziale negli spazi metrico-projettivi, Memorie R. Accad. Sci. Lett. Modena, III, VIII (Appendice) (1907), 21–66.

Planar graphs: one graph to rule them all

Marthe Bonamy

(based on joint work with Cyril Gavoille and Michał Pilipczuk)

Consider all planar graphs on n vertices. What is the smallest graph that contains them all as induced subgraphs? We provide an explicit construction of such a graph of size $n^{\frac{4}{3}+o(1)}$ [1], which improves upon the previous best upper bound of $n^{2+o(1)}$ [3].

In this talk, we will gently introduce the audience to the notion of so-called universal graphs (graphs containing all graphs of a given family as induced subgraphs), and devote some time to a key lemma in the proof. That lemma comes from a recent breakthrough [2] regarding the structure of planar graphs, and has already many interesting consequences - we hope the audience will be able to derive more.

References

[1] M. Bonamy, C. Gavoille, M. Pilipczuk, Shorter labeling schemes for planar graphs, arXiv:1908.03341 (2019).

[2] V. Dujmović, G. Joret, P. Micek, P. Morin, T. Ueckerdt, D. Wood, Planar graphs have bounded queue-number, arXiv:1904.04791 (2019).

[3] C. Gavoille, A. Labourel, Shorter implicit representation for planar graphs and bounded treewidth graphs, in: L. Arge, M. Hoffmann, W. Welzl (eds.), Algorithms ESA 2007, LNCS 4698 (2007), 582–593.

Bipartite graphs with close domination and k-domination numbers

Csilla Bujtás

(joint work with Gülnaz Boruzanlı Ekinci)

Let k be a positive integer and let G be a graph with vertex set $V(G)$. A subset $D \subseteq V(G)$ is a k-dominating set if every vertex outside D is adjacent to at least k vertices in D. The k-domination number $\gamma_k(G)$ is the minimum cardinality of a k-dominating set in G. It was proved in [3] that $\gamma_k(G) \geq \gamma(G) + k - 2$ holds for every graph G with $\Delta(G) \geq k \geq 2$ and this bound is sharp for every $k \geq 2$.

In an earlier work [1] we studied graphs satisfying $\gamma_2(G) = \gamma(G)$. In this talk, based on [2], we characterize bipartite graphs satisfying the equality $\gamma_k(G) = \gamma(G) + k - 2$ for each $k \geq 3$. While studying the problem, we introduce the notion of 'underlying hypergraph' and also a new hypergraph invariant which is called vertex-edge cover number. Further, we identify those bipartite graphs which satisfy the equality $\gamma_3(G) = \gamma(G) + 1$ hereditarily. It is also proved that the problem of deciding whether a graph satisfies the given equality is NP-hard for each $k \geq 2$.

References

[1] G. Boruzanlı Ekinci, Cs. Bujtás, On the equality of domination number and 2-domination number, arXiv:1907.07866 (2019).
[2] G. Boruzanlı Ekinci, Cs. Bujtás, Bipartite graphs with close domination and k-domination numbers, manuscript (2019).
[3] J.F. Fink, M.S. Jacobson, n-domination in graphs, Graph theory with applications to algorithms and computer science (1985), 283–300.

Light edges in the class of toroidal graphs

Katarína Čekanová

(joint work with Mária Maceková and Roman Soták)

Let $\mathcal{G}$ be a class of graphs. The weight $w(e)$ of an edge e is the sum of the degrees of its endvertices. We say that edge is *light* in the class $\mathcal{G}$ if there exists a constant k such that every graph $G \in \mathcal{G}$ contains an edge with $w(e) \leq k$. Kotzig proved that every 3-connected plane graph contains an edge with weight at most 13. Borodin extended this result to normal plane maps and Jendrol' described the exact types of edges in such graphs.

Plane graphs with minimum degree 2 do not necessarily contain a light edge (e.g. $K_{2,r}$, for $r \geq 2$). However, if we consider additional condition on the girth $g(G)$ of the graph (the length of the shortest cycle in G), then G will contain an edge of weight at most 7 for $g(G) \geq 5$.

Light edges in the class of graphs embeddable on the surfaces with higher genus were investigated by Ivančo; later Jendrol', Tuhársky and Voss described exact types of edges in large maps on surfaces with $\delta(G) \geq 3$.

In this talk we describe exact types of edges in connected toroidal graphs with minimum degree 2 and girth at least 4.

Finding longest paths in graphs associated with hybrid wireless sensor networks

Chiuyuan Chen

(joint work with Chao-Wei Chen and Wu-Hsiung Lin)

Given a graph $G = (V, E)$ and a positive integer $K \leq |V| - 1$, the *longest path* problem is to determine if G contains a simple path (that is, a path going through no vertex more than once) with K or more edges. A *split graph* is a graph in which its vertices can be partitioned into a clique and an independent set. It is known that the longest path problem is NP-complete and remains NP-complete for split graphs.

In this talk, we focus on a subclass Γ of split graphs originated from the problem of maximizing the lifetime of barrier coverage in a hybrid sensor network, which consists of weak (energy-limited) static sensors and strong (energy-rechargeable) mobile sensors. Reference [1] formulates the maximum lifetime barrier-coverage in hybrid sensor network problem as the longest path problem in Γ and proposes a polynomial-time algorithm to find such a path. However, we find that the algorithm in [1] does not always produce a longest path. We therefore try to propose a correct algorithm.

References

[1] D. Kim, W. Wang, J. Son, W. Wu, W. Lee, A.O. Tokuta, Maximum lifetime combined barrier-coverage of weak static sensors and strong mobile sensors, IEEE Trans. Mobile Comput. 16 (2017), 1956–1966.

Regular locally hamiltonian graphs

Johan P. de Wet

(joint work with Marietjie Frick)

We say a graph G is locally hamiltonian if $\langle N(v)\rangle$, the graph induced by the neighbourhood of v, is hamiltonian for any $v \in V(G)$. In 1983 Pareek and Skupień [2] asked whether there exist any connected, locally hamiltonian graphs that are r-regular and nonhamiltonian. At the Cycles and Colourings workshop of 2018, Nedela [1] asked whether every r-regular triangulation on a surface is hamiltonian. Since a triangulation of a surface is locally hamiltonian, these two problems are closely related. We answer the first question by showing how to construct connected r-regular locally hamiltonian graphs that are not hamiltonian for $r \geq 11$. We also show that the Hamilton Cycle Problem for such graphs is NP-complete. The second question is more difficult, but we show how the techniques developed for locally hamiltonian graphs can be applied to triangulations on surfaces. We conjecture that there do exist r-regular nonhamiltonian triangulations on a surface.

References

[1] R. Nedela, Problems listed for Cycles and Colourings, 2018, https://candc.upjs.sk/history/18problems.pdf

[2] C.M. Pareek, Z. Skupień, On the smallest non-Hamiltonian locally Hamiltonian graph, J. Univ. Kuwait (Sci.) 10 (1983), 9–17.

On cycle-antimagic labelings

Andrea Feňovčíková
(joint work with Martin Bača, P. Jeyanthi,
N.T. Muthuraja, and Pothukutti Nadar Selvagopal)

A simple graph G admits an H-covering if every edge in $E(G)$ belongs to a subgraph of G isomorphic to H. An (a, d)-H-antimagic total labeling of a graph G admitting an H-covering is a bijective function from the vertex set $V(G)$ and the edge set $E(G)$ of the graph G onto the set of integers $\{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that for all subgraphs H' isomorphic to H, the sum of labels of all the edges and vertices belonged to H' constitute the arithmetic progression with the initial term a and the common difference d. Such a labeling is called *super* if the smallest possible labels appear on the vertices.

In this talk, we will deal with the existence of the super (a, d)-H-antimagic total labelings of wheels, fan graphs and ladders for H isomorphic to a cycle.

The strong path partition conjecture

Marietjie Frick
(joint work with Johan de Wet, Jean Dunbar, and Ortrud Oellermann)

The number of vertices in a longest path in a graph G is denoted by $\tau(G)$. Let (a, b) be an arbitrary pair of positive integers. If the vertex set of a graph G can be partitioned into two sets A and B such that

$$\tau(\langle A \rangle) \leq a \text{ and } \tau(\langle B \rangle) \leq b,$$

we say that (A, B) is an (a, b)*-partition* of G. If equality holds in both instances, then (A, B) is an *exact* (a, b)*-partition*.

The *Path Partition Conjecture*(*PPC*) asserts that if G is any graph such that $a + b = \tau(G)$, then G has an (a, b)-partition. The *Strong PPC* asserts that under the same circumstances G has an exact (a, b)-partition.

The PPC is a long-standing conjecture. Since its first appearance in the literature (in [2]), several results in support of this conjecture have been proved (see [1] for a summary). However, the partitioning techniques used in those proofs have turned out to be unsuitable for producing exact partitions.

The PPC is known to hold for $a \leq 8$ (see [3]). In this talk it will be shown that the Strong PPC holds for $a \leq 7$.

References

[1] J.E. Dunbar, M. Frick, The path partition conjecture, in: R. Gera, T.W. Haynes, S.T. Hedetniemi (eds.), Graph Theory: Favorite Conjectures and Open Problems, pp. 101–113, Springer 2018.

[2] J.M. Laborde, C. Payan, N.H. Xuong, Independent sets and longest directed paths in digraphs, in: Graphs and other combinatorial topics, Proc. 3rd Czech. Symp., Prague 1982, Teubner-Texte Math. 59 (1983), 173–177.

[3] L.S. Mel'nikov, I.V. Petrenko, Path kernels and partitions of graphs with small cycle length, in: V.N. Kasyanov (ed.) Methods and tools of program construction and optimization, ISI SB Russian Academy of Science, Novosibirsk (2005), 145–160 (in Russian).

One edge hamiltonian graphs

Grzegorz Gancarzewicz

We consider only finite graphs without loops and multiple edges. Using the generalized Ore's condition given by Nicolas Lichiardopol [1] we will give a sufficient condition under which a graph is 1-edge hamiltonian or hamiltonian connected.

References

[1] N. Lichiardopol, New Ores type results on hamiltonicity and existence of paths of given length in graphs, Graphs Combin. 29 (2013), 99–104.

The canonical Tutte polynomial for signed graphs

Andrew Goodall

(joint work with Guus Regts, Lluis Vena, and Bart Litjens)

The "trivariate Tutte polynomial" of a signed graph, newly discovered by Goodall, Litjens, Regts and Vena [1], contains among its evaluations both the number of proper colorings of a signed graph (enumerated by Zaslavsky decades ago) and the number of nowhere-zero flows (only recently enumerated by Qian and independently by Goodall, Litjens, Regts and Vena). In this, the trivariate Tutte polynomial parallels the Tutte polynomial of a graph, which contains the chromatic polynomial and flow polynomial as specializations. The resemblances do not end there, and in this talk I will sketch why this polynomial invariant of signed graphs merits the title of "the canonical Tutte polynomial".

References

[1] A.J. Goodall, B.M. Litjens, G. Regts, L. Vena, Tutte's dichromate for signed graphs, arXiv:1903.07548 (2019).

Cyclewidth: A branch decomposition for directed graphs

Meike Hatzel
(joint work with Archontia Giannopoulou,
Roman Rabinovich, and Sebastian Wiederrecht)

Cyclewidth is a new measure for directed graphs. It is qualitatively equivalent to directed treewidth, but it comes with somewhat nicer properties. For example it is closed under taking butterfly minors. The decomposition is a branch decomposition which makes it easier to relate to other width measures as well. As a consequence we are able to give a nice characterisation of digraphs of cyclewidth one, a question that is still open for directed treewidth. Cyclewidth also acts as a connection between the perfect matching width of bipartite graphs with perfect matchings and the directed treewidth of directed graphs. Thus it allows for the translation of structural results between directed graphs and bipartite matching covered graphs which we illustrate on the example of a grid theorem.

Constructing a uniquely Hamiltonian infinite graph all whose vertices and ends have degree 3

Karl Heuer

By a result of Smith appearing in a paper of Tutte [5] we know that no finite cubic graph exists which is uniquely Hamiltonian, i.e. contains precisely one Hamilton cycle.

In order to address questions regarding Hamiltonicity in infinite graphs, we follow a topological approach introduced by Diestel and Kühn [1, 2] for defining infinite cycles. For a locally finite connected graph G we define its infinite cycles via its Freudenthal compactification $|G|$ as follows: We call a homeomorphic image of the unit circle $S^1 \subseteq \mathbb{R}^2$ in $|G|$ an *infinite cycle of G*. Consequently, we call an infinite cycle of G containing all vertices of G a *Hamilton cycle of G*.

The compactification points of G in its Freudenthal compactification $|G|$ can be understood in a purely combinatorial way: First we define an equivalence relation on the set of all one-way infinite paths in G by declaring two such paths *equivalent* if they are joined by infinitely many vertex-disjoint paths in G. The equivalence classes of this relation are called the *ends* of G, and those correspond precisely to the compactification points of G in $|G|$. For an end ω of G we now define its degree $d(\omega)$ as follows:

$$d(\omega) := \sup\{|\mathcal{R}| \; ; \; \mathcal{R} \text{ is a set of disjoint one-way infinite paths in } \omega\}.$$

Mohar [4] asked the question whether a uniquely Hamiltonian infinite graph exists in which every vertex and every end have the same degree d for some $d \in \mathbb{N}$.

I [3] answered this question positively by giving a construction for $d = 3$, which is in contrast to the result of Smith for finite graphs mentioned above.

In this talk I will present this construction.

References

[1] R. Diestel, D. Kühn, On infinite cycles I, Combinatorica 24 (2004), 69–89.

[2] R. Diestel, D. Kühn, On infinite cycles II, Combinatorica 24 (2004), 91–116.

[3] K. Heuer, Hamiltonicity in locally finite graphs: two extensions and a counterexample, Electron. J. Comb. 25 (2018), P3.13.

[4] B. Mohar, http://www.fmf.uni-lj.si/~mohar/Problems/P0703_Hamiltonicity Infinite.html.

[5] W.T. Tutte, On Hamiltonian circuits, J. London Math. Soc. 21 (1946), 98–101.

Induced path numbers of regular graphs

Daniel Horsley

(joint work with Saieed Akbari and Ian Wanless)

The *path cover number* of a graph G, the smallest size of a collection of paths in G such that every vertex of G is in exactly one of the paths, has been very well studied. The *induced path number*, the analogous quantity when we also demand that the paths be induced, has also received some attention. This talk will discuss some bounds on the induced path number of connected regular graphs that we established recently, focussing on the cubic case.

Counting double covers of planar graphs

Radek Hušek

(joint work with Peter Korcsok and Robert Šámal)

Several recent results and conjectures study counting versions of classical existence statements. Esperet et al. [3] proved Lovász–Plummer conjecture: every bridgeless cubic graph has exponentially many perfect matchings. Thomassen proved that every planar graph has exponentially many (list) 5-colorings.

Thomassen [4] also asked the same question for 3-colorings of triangle-free planar graphs. He gave a subexponential bound that was later improved by Asadi et al. [1]. However, the conjecture stays open. By duality, these results and conjecture can be equivalently stated for the number of nowhere-zero $\mathbb{Z}_5$-flows (or $\mathbb{Z}_3$-flows) of planar (4-edge-connected) graphs. Dvořák, Mohar and Šámal [2] prove that the number of $\mathbb{Z}_2 \times \mathbb{Z}_3$-flows in a 3-edge-connected graphs is at least $2^{n/7}$. They provide exponential bounds also for $\mathbb{Z}_2 \times \mathbb{Z}_2$-flows and $\mathbb{Z}_3$-flows.

We ask the same question for cycle double covers of cubic graphs. We show that counting cycle double covers usually allows "cheating" by splitting a cycle consisting of more circuits into many cycles, and for this reason we try to count circuit double covers instead. We give an almost exponential bound for planar graphs:

Theorem. *Every bridgeless cubic planar graph $G = (V, E)$ has at least $2^{\Theta(\sqrt[4]{|V|})}$ circuit double covers.*

References

[1] A. Asadi, Z. Dvořák, L. Postle, R. Thomas, Sub-exponentially many 3-colorings of triangle-free planar graphs, J. Combin. Theory Ser. B 103 (2013), 706–712.
[2] Z. Dvořák, B. Mohar, R. Šámal, Exponentially many nowhere-zero $\mathbb{Z}_3$-, $\mathbb{Z}_4$-, and $\mathbb{Z}_6$-flows, arXiv:1708.09579 (2017).
[3] L. Esperet, F. Kardoš, A.D. King, D. Kráľ, S. Norine, Exponentially many perfect matchings in cubic graphs, Adv. Math. 227 (2011), 1646–1664.
[4] C. Thomassen, Many 3-colorings of triangle-free planar graphs, J. Combin. Theory Ser. B 97 (2007), 334–349.

On the cyclic coloring conjecture

Stanislav Jendroľ

(joint work with Roman Soták)

A cyclic coloring of a plane graph G is a coloring of its vertices such that vertices incident with the same face have distinct colors. The minimum number of colors in a cyclic coloring of a plane graph G is its cyclic chromatic number $\chi_c(G)$. Let $\Delta^*(G)$ be the maximum face degree of a graph G and $t(G)$ denotes the order of a longest path of G all vertices of which are of degree 2. For a 2-connected plane graph G let $R(G)$ be the graph (called the reduction of G) obtained from G by replacing all maximal paths all interior vertices of which have degree 2 with edges.

We show that The Cyclic Coloring Conjecture of Borodin from 1984, saying that every connected plane graph G has $\chi_c(G) \leq \lfloor \frac{3}{2}\Delta^*(G) \rfloor$, can be reduced to hold for 2-connected plane graphs G whose reductions $R(G)$ are simple 3-connected plane graphs. We have received three different upper bounds for graphs G from this restricted family. Moreover, we have proved that the conjecture of Borodin holds for 2-connected plane graphs with a large maximum face degree and for two wide families of plane graphs.

Packing and covering directed triangles asymptotically

Adam Kabela

(joint work with Jacob Cooper, Andrzej Grzesik, and Dan Kráľ)

We show that every digraph D on n nodes contains a set T of arc-disjoint directed triangles and a set E of arcs so that $D - E$ has no directed triangles and $\frac{9}{5}|T| \geq |E| - o(n^2)$.

The study is motivated by the question of Tuza [2] asking about such sets T and E and the smallest constant c satisfying $c|T| \geq |E|$, and by the conjecture of McDonald et al. [1] suggesting that $c \leq \frac{3}{2}$ for every directed multigraph.

References

[1] J. McDonald, G.J. Puleo, C. Tennenhouse, Packing and covering directed triangles, arXiv:1806.08809 (2018).

[2] Zs. Tuza, A conjecture on triangles of graphs, Graphs Combin. 6 (1990), 373–380.

Colouring Schrijver graphs: From combinatorics to topology and back again

Tomáš Kaiser

(joint work with Matěj Stehlík)

We will discuss ‘topological’ approaches to determining the chromatic number of Kneser graphs and Schrijver graphs, starting with the breakthrough proof of the Lovász–Kneser theorem in 1978.

Moving on to recent developments in this direction, we will recall the notion of projective quadrangulation which provides a geometric perspective on the problem. The class of projective quadrangulations includes many known graphs (including all generalised Mycielski graphs), and results on their chromatic number can be obtained as consequences of a general lower bound to the chromatic number of projective quadrangulations.

While Schrijver graphs (as well as Kneser graphs) are usually not projective quadrangulations, we will show that each Schrijver graph contains a spanning projective quadrangulation with the same chromatic number, obtaining an alternative proof of the Lovász–Kneser theorem.

Although this proof uses topological tools (essentially the Borsuk–Ulam theorem) just like the original proof of Lovász, we will discuss a fairly direct combinatorial formulation using a form of Fan's lemma, found by Müller and Stehlík.

We will then turn our attention to the question whether the above projective quadrangulations might in fact be edge-critical subgraphs of Schrijver graphs with the same chromatic number, much like Schrijver graphs themselves are vertex-critical subgraphs of Kneser graphs. In some cases at least, the answer is known to be affirmative, and we will present appealing combinatorial descriptions of these edge-critical subgraphs which can be read off from their geometric construction.

On 2-distinguishable graphs

Rafał Kalinowski

(joint work with Wilfried Imrich, Monika Pilśniak, and Mariuzs Woźniak)

A vertex colouring of a graph G is called *asymmetric* if the identity is the only automorphism preserving the colouring. A graph is *d-distinguishable* if G admits an asymmetric colouring with d colours. We focus on 2-distinguishable graphs. The *motion of a graph G* is the least number $m(G)$ of vertices moved by every non-identity automorphism of G.

A year ago, László Babai [1] asked the following question: does there exist a function $f(d)$ such that every connected, countable graph G with maximum degree $\Delta(G) \leq d$ and motion $m(G) > f(d)$, is 2-distinguishable? The value of $f(3)$ was determined in [2]. In [3], it was shown that $f(d) = 2\lceil \log_2 d \rceil$ for trees. In this talk, we investigate $f(4)$ for claw-free graphs.

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On the 4-color theorem for signed graphs

František Kardoš

(joint work with Jonathan Narboni)

There are several ways to generalize graph coloring to signed graphs. Máčajová, Raspaud and Škoviera introduced one of them and conjectured that in this setting, for signed planar graphs four colors are always enough, generalising thereby The Four Color Theorem. In this talk, we disprove this conjecture.

On the chromatic edge stability index of graphs

Arnfried Kemnitz
(joint work with Massimiliano Marangio)

The χ'-edge stability number or chromatic edge stability index $es_{\chi'}(G)$ of a graph G is the minimum number of edges of G whose removal results in a graph $H \subseteq G$ with $\chi'(H) = \chi'(G) - 1$ ($es_{\chi'}(G) = 0$ if $E(G) = \emptyset$).

In this talk, we give some general results for the chromatic edge stability index, such as lower and upper bounds. Moreover, we determine $es_{\chi'}(G)$ exactly for several well-known classes of graphs.

Comparing Graovac-Pisanski index with Wiener index

Martin Knor
(joint work with Riste Škrekovski and Aleksandra Tepeh)

Graovac-Pisanski index, originally called a modified Wiener index, combines the distances of a graph with its automorphisms. Let G be a graph. Its Graovac-Pisanski index, $\mathrm{GP}(G)$, is defined as

$$\mathrm{GP}(G) = \frac{|V(G)|}{2|\mathrm{Aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \mathrm{Aut}(G)} \mathrm{dist}(u, \alpha(u)),$$

where $\mathrm{Aut}(G)$ is the group of automorphisms of G and $\mathrm{dist}(u,v)$ is the distance from u to v in G. For any $S \subseteq V(G)$ we have $W_G(S) = \sum_{u,v \in S} \mathrm{dist}(u,v)$. Then $W_G(V(G))$ is the famous Wiener index. Moreover, $\mathrm{GP}(G) = |V(G)| \cdot \sum_{i=1}^{t} \frac{W_G(V_1)}{|V_i|}$, where $V_1, V_2, \dots, V_t$ are the orbits of $\mathrm{Aut}(G)$. Hence, $\mathrm{GP}(G) = 0$ if all orbits consist of single vertices, that is if $\mathrm{Aut}(G)$ contain only the identity. On the other hand, if G is vertex-transitive, that is if there is just one orbit of $\mathrm{Aut}(G)$, then $\mathrm{GP}(G) = W(G)$. One can conclude that $0 \leq \mathrm{GP}(G) \leq W(G)$, and indeed, the first inequality is trivial. We show that the second inequality holds for trees, but there are graphs G such that $\mathrm{GP}(G) > W(G)$, and there are graphs G' which are not vertex-transitive and though $\mathrm{GP}(G') = W(G')$. If T is a tree, then $\mathrm{GP}(T)$ is an integer number. We show that $W(T) - \mathrm{GP}(T) = c$ holds for some tree T if and only if c is a nonnegative integer different from 3 and 4.

Equitable colorings on shifted toroidal grids

Wu-Hsiung Lin
(joint work with Kuo-Ching Huang and Hau-Yi Lin)

An equitable k-coloring of a graph G is a mapping $f : V(G) \to \{1, 2, \dots, k\}$ such that $f(x) \neq f(y)$ for $xy \in E(G)$ and $\big||f^{-1}(i)| - |f^{-1}(j)|\big| \leq 1$ for $1 \leq i < j \leq k$. The equitable chromatic number $\chi_=(G)$ is the smallest integer k for which G admits an equitable k-coloring, and the equitable chromatic threshold $\chi_=^*(G)$ is the smallest integer t for which G admits an equitable k-coloring for $k \geq t$. Note that $\chi(G) \leq \chi_=(G) \leq \chi_=^*(G)$. The toroidal grid (grid on a torus) $T_{m,n}$ is isomorphic to $C_m \square C_n$, and the shifted toroidal grid $T_{m,n}^i$ is the i-layer-shifted grid on a torus. The circulant graphs $C_n(a_1, a_2, \dots, a_m)$ consists of vertex set $\{0, 1, \dots, n-1\}$ and edge set $\big\{ij : \min\{|i-j|, n-|i-j|\} \in \{a_1, a_2, \dots, a_m\}\big\}$. It is shown that $T_{m,n}^i$ is isomorphic to $C_{mn}(a, b)$ [2], and $2 \leq \chi(C_n(a, b)) \leq 5$ has been completely characterized [1].

We investigate the equitable colorability and the condition of which a, b, n the equalities $\chi(C_n(a, b)) = \chi_=(C_n(a, b)) = \chi_=^*(C_n(a, b))$ hold for connected $C_n(a, b)$. We verify the equitable colorability of all $C_n(a, b)$ with chromatic number 4 or 5 and all $C_n(a, n/2)$, and we propose equitable 3-colorings for bipartite $C_n(a, b)$ when n is large. We also examine that the inequality is sharp for $C_n(1, 3)$ when $n = 6, 8, 10, 12$, and verify the equitable 3-colorability of $C_n(a, b)$ for $n \leq 16$.

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Tilings of the sphere by almost equilateral pentagons

Hoi Ping Luk
(joint work with Min Yan)

The classification of edge-to-edge tilings of the sphere by congruent pentagons can be divided into three cases: variable edge lengths, equilateral, and almost equilateral. The first two cases have been largely settled by Min Yan and his collaborators. The almost equilateral case (four edges of the same length and the fifth different) is the most difficult one, and earlier techniques are insufficient. We have introduced decision-making algorithms in wxMaxima and new geometric constraints to handle this case. We have obtained full classification for almost equilateral pentagons with three distinct angles and partial results for those with five distinct angles. We will discuss our findings which include Earth Map Tilings and some special tilings not seen in the other pentagon cases.

A 3-edge-colouring algorithm

Robert Lukoťka

(joint work with Jakub Tětek)

Let G be a subcubic graph and let D be a path decomposition of G. We present an algorithm that decides whether G is 3-edge-colourable with running time $O(|V(G)| \cdot 3^{w(D)})$, where $w(D)$ denotes the width of D. Using the path-width algorithm of Fomin and Høie [1] we get an algorithm with running time $O(3^{(1/6+\varepsilon)\cdot|V(G)|})$, for an arbitrary $\varepsilon > 0$, that is $O(1.201^{|V(G)|})$. This is a significant improved over the until now asymptotically best algorithm by Kowalik [2], which runs in $O(1.344^{|V(G)|})$. The algorithm is (with some slight modifications) easy to implement and practical. The method can also be used to calculate several related invariants for cubic graphs. We present modifications of the algorithm that calculate resistance, weak oddness, and strong oddness.

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Between proper and strong edge-colorings of subcubic graphs

Borut Lužar

(joint work with Herve Hocquard and Dimitri Lajou)

In the talk, we will discuss edge-colorings of (sub)cubic graphs. Namely, we will consider edge-colorings in which we work with two types of colors: the *proper* and the *strong colors*. The edges colored with the same proper color form a matching (no two edges are incident), and the edges colored with the same strong color form an induced matching (no two edges are incident to a common edge). Clearly, when all the colors are proper, the edge-coloring of a graph is proper, and if all the colors are required to be strong, we have a strong edge coloring. The tight upper bounds for the chromatic indices of the above two extremal colorings are long established, thus we will focus to edge-colorings with combinations of proper and strong colors. Such colorings have been investigated before, but only as a tool to obtain results for other types of colorings. Systematically, they have been introduced just recently by Gastineau and Togni as the edge coloring variation of S-packing colorings [1]. We will present results on the topic and give a number of open problems.

References

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Packing directed circuits quarter-integrally

Tomáš Masařík

(joint work with Irene Muzi, Marcin Pilipczuk, Paweł Rzążewski, and Manuel Sorge)

The celebrated Erdős-Pósa theorem [1] states that every undirected graph that does not admit a family of k vertex-disjoint cycles contains a feedback vertex set (a set of vertices hitting all cycles in the graph) of size $O(k \log k)$. After being known for long as Younger's conjecture, a similar statement for directed graphs has been proven in 1996 by Reed, Robertson, Seymour, and Thomas [2]. However, in their proof, the dependency of the size of the feedback vertex set on the size of vertex-disjoint cycle packing is not elementary.

We show that if we compare the size of a minimum feedback vertex set in a directed graph with *quarter-integral* cycle packing number, we obtain a polynomial bound. More precisely, we show that if in a directed graph G there is no family of k cycles such that every vertex of G is in at most *four* of the cycles, then there exists a feedback vertex set in G of size $O(k^4)$. On the way there we prove a more general result about quarter-integral packing of subgraphs of high directed treewidth: for every pair of positive integers a and b, if a directed graph G has directed treewidth $\Omega(a^6 b^8 \log^2(ab))$, then one can find in G a family of a subgraphs, each of directed treewidth at least b, such that every vertex of G is in at most four subgraphs.

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On uniquely colourable graphs

Samuel Mohr

A uniquely k-colourable graph is a graph with exactly one partition of the vertex set into k colour classes. Here, we investigate some constructions of uniquely k-colourable graphs and give a construction of K_k-free uniquely k-colourable graphs with equal colour class sizes.

Coloring triangulations, even triangulations and quadrangulations on surfaces

Atsuhiro Nakamoto

Four Color Theorem is one of the most celebrated results on graph coloring, but its proof is very complicated. On the other hand, we can easily verify that every *even* triangulation on the plane (i.e., one with each vertex of even degree) is 3-colorable, and that every *quadrangulation* (i.e., one with each face quadrilateral) on the plane is 2-colorable.

If we focus on a *locally planar graph* on a non-spherical orientable surface (i.e., a graph on the surface with sufficiently large width), then the following are known: (i) every triangulation is 5-colorable, (ii) every even triangulation is 4-colorable, and (iii) every quadrangulation is 3-colorable, where those numbers are best possible. However, if we consider the same problems for nonorientable surfaces, then the triple $(5, 4, 3)$ of the upper bounds of chromatic numbers in (i), (ii), (iii) changes into $(5, 5, 4)$, and this strange fact can be explained by the existence of "odd quadrangulations" on nonorientable surfaces.

In my talk, dealing with chromatic numbers of triangulations, even triangulations and quadrangulations on surfaces, we report on the above results and related topics. We also mention our recent progress on this area of researches.

Harnessing the power of atoms

Jana Novotná

(joint work with Konrad Dąbrowski, Tomáš Masařík,
Daniël Paulusma, and Paweł Rzążewski)

A graph is H-free if it does not contain the graph H as an induced subgraph. Recently, a detailed study about clique-width of (H_1, H_2)-free graphs has been published. There, the classes have been classified based on the boundedness of clique-width and for only a few cases the bounds still remain unknown. It is well known that for graphs of bounded clique-width exists an effective algorithm for solving many graph problems. Some problems of combinatorial optimization can be solved effectively even on a larger class of graphs. Michaël Rao shows an algorithm for graphs that are composed of atoms of small clique-width glued together by clique cut sets, where *atom* is an induced subgraph that does not contain a clique cut set. Using clique cut set the solution of some problems can be inductively composed out of atoms, and therefore, we get an effective algorithm for e.g., graph coloring or independent set.

If we look at graphs with only one forbidden induced subgraph, it is well known that a transition to atoms does not help. Although, for two forbidden induced subgraphs, Gaspers, Huang, and Paulusma showed the first positive result for (C_4, P_6)-free graphs, i.e., those graphs have unbounded clique-width but bounded clique-width of atoms. We extend the study of clique-width and for most of the classes of graphs with two forbidden induced subgraphs and unbounded clique-width, we determine whether the clique-width of their atoms is bounded or not. For most classes, the transition to atoms does not help. We are able to modify existing constructions or derive new constrictions showing unboundedness of clique-width even for atoms. On the other hand, we provide one another positive evidence for (Triplet, $2P2$)-free graphs, where Triplet is C_4 with one additional vertex that is adjacent to exactly three vertices of C_4.

Generalized coloring of permutations

Michal Opler

(joint work with Vít Jelínek and Pavel Valtr)

A permutation π is a *merge* of a permutation σ and a permutation τ, if we can color the elements of π red and blue so that the red elements have the same relative order as σ and the blue ones as τ. We consider, for fixed hereditary permutation classes $\mathcal{C}$ and $\mathcal{D}$, the complexity of determining whether a given permutation π is a merge of an element of $\mathcal{C}$ with an element of $\mathcal{D}$.

We develop general algorithmic approaches for identifying polynomially tractable cases of merge recognition. Our tools include a version of nondeterministic logspace streaming recognizability of permutations, which we introduce, and a concept of bounded width decomposition, inspired by the work of Ahal and Rabinovich [1].

As a consequence of the general results, we can provide nontrivial examples of tractable permutation merges involving commonly studied permutation classes, such as the class of layered permutations, the class of separable permutations, or the class of permutations avoiding a decreasing sequence of a given length.

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The size-Ramsey number of tight 3-uniform paths

Olaf Parczyk

(joint work with Jie Han, Yoshiharu Kohayakawa, and Guilherme Oliveira Mota)

Given a hypergraph H, the size-Ramsey number $\hat{r}_2(H)$ is the smallest integer m such that there exists a hypergraph G with m edges with the property that in any colouring of the edges of G with two colours there is a monochromatic copy of H. We prove that the size-Ramsey number of the 3-uniform tight path on n vertices $P_n^{(3)}$ is linear in n, i.e., $\hat{r}_2(P_n^{(3)}) = O(n)$. This answers a question by Dudek, La Fleur, Mubayi, and Rödl for 3-uniform hypergraphs [1].

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χ-boundedness for limited induced odd-cycle packing

Jakub Pekárek

(joint work with Zdeněk Dvořák)

The graph classes defined by variously forbidden cycles figure prominently in the structural graph theory. In coloring problems such classes are motivated in particular by the Strong Perfect Graph Theorem and the concept of χ-boundedness. A class of graphs is χ*-bounded* if the chromatic number of the graphs from the class can be bounded by a function of the clique number.

Motivated by recent algorithmic results exploiting the property of not having two disjoint odd cycles (satisfied by certain geometric graph classes) we study the chromatic number of k-OC-free graphs, defined as graphs containing no induced subgraph consisting of k pairwise vertex-disjoint odd cycles.

We show that if G is k-OC-free of girth $g \geq 4$, then $\chi(G) \leq 5k - 2$ and furthermore, if $g \geq 7$, then $\chi(G) \leq k + 2$. In general case, every k-OC-free graph G has chromatic number at most $f(k, \omega(G))$ for some function $f(k, \omega) = O(\omega^{3k-3})$. On the other hand, a standard probabilistic construction shows that for every integer $\omega \geq 1$, there exists a 2-OC-free graph G_ω with clique number at most ω s.t. $\chi(G) \geq \Theta(\omega^2 / \log^2 \omega)$.

Short rainbow cycles and cuts

Petra Pelikánová

(joint work with Aneta Šťastná and Sophie Spirkl)

Rainbow cycle in edge-colored graph is a cycle with all edges of distinct colors. Aharoni conjecture about short rainbow cycles in edge-colored graphs is generalization of Caccetta-Häggkvist conjecture [2].

Conjecture (Aharoni [1]). *Let G be a simple n-vertex graph and c be a coloring of $E(G)$ with n colors, where each color class has size at least k. Then G contains a rainbow cycle of length at most $\lceil \frac{n}{k} \rceil$.*

The conjecture was proved for $k = 2$ by DeVos et al [3]. This result guarantees existence of rainbow cycle C_l of length at most $\lceil \frac{n}{2} \rceil$ for bigger k. The talk is focused on structure of cuts between this rainbow cycle C_l and the rest of the graph. There will be introduced cuts which implies short rainbow cycles for $k = 3$.

References

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Asymmetric colourings of infinite graphs

Monika Pilśniak

A colouring of a graph G is called *asymmetric* if the identity is the only automorphism preserving the colouring. The *distinguishing number* $D(G)$ of a graph is the least number of colours in an asymmetric vertex colouring. It was introduced by Albertson and Collins in [1], and was first considered for infinite graphs by Imrich, Klavžar and Trofimov in [3]. An asymmetric edge colourings was first investigated by Kalinowski and Pilśniak in [4] and for infinite graphs by Broere and Pilśniak [2].

In the talk, we survey results on asymmetric vertex and edge colourings of infinite graphs. We give known general upper bounds in terms of the maximum degree. We focus mainly on several classes of graphs which need only two or three colours to break all nontrivial automorphisms. The most intriguing conjecture in this area is an Infinite Motion Conjecture posed by Thomas Tucker that every connected locally finite graph, such that every nontrivial automorphism moves infinitely many vertices, has the distinguishing number at most two. We show very recent results obtained together with Lehner and Stawiski in this topic.

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Hamiltonian problems in line graphs of 3-hypergraphs

Zdeněk Ryjáček

(joint work with Binlong Li, Kenta Ozeki, and Petr Vrána)

Line graphs of 3-hypergraphs can be considered as a natural generalization of line graphs of (multi-)graphs (note that a line graph of a 3-hypergraph is $K_{1,4}$-free but can contain induced claws $K_{1,3}$). We extend some statements related to a 1986 conjecture by Thomassen (every 4-connected line graph is hamiltonian) to this class, and we generalize them to Tutte cycles and paths (a cycle/path is Tutte if any component of its complement has at most three vertices of attachment).

Among others, we formulate the following conjectures:

(i) every 2-connected line graph of a 3-hypergraph has a Tutte maximal cycle containing any two prescribed vertices,

(ii) every 3-connected line graph of a 3-hypergraph has a Tutte maximal cycle containing any three prescribed vertices,

(iii) every connected line graph of a 3-hypergraph has a Tutte maximal (a, b)-path for any two vertices a, b,

(iv) every 4-connected line graph of a 3-hypergraph is Hamilton-connected,

and we show that all these (seemingly much stronger) statements are still equivalent with Thomassen's conjecture.

Polynomial χ-binding functions for P_5-free graphs

Ingo Schiermeyer

(joint work Christoph Brause and Maximilian Geißer)

A graph G with clique number $\omega(G)$ and chromatic number $\chi(G)$ is *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H of G. A family $\mathcal{G}$ of graphs is called χ-bounded with binding function f if $\chi(G') \leq f(\omega(G'))$ holds whenever $G \in \mathcal{G}$ and G' is an induced subgraph of G.

In this talk we will present a survey on polynomial χ-binding functions. Especially we will address perfect graphs, hereditary graphs satisfying the Vizing bound ($\chi \leq \omega + 1$), graphs having linear χ-binding functions and graphs having non-linear polynomial χ-binding functions. Thereby we also survey polynomial χ-binding functions for several graph classes defined in terms of forbidden induced subgraphs, among them $2K_2$-free graphs, P_k-free graphs, *claw*-free graphs, and *diamond*-free graphs. Our main focus will be on recent results for P_5-free graphs.

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On DP-coloring of digraphs

Thomas Schweser

(joint work with Jørgen Bang-Jensen, Thomas Bellitto, and Michael Stiebitz)

DP-coloring is a relatively new coloring concept by Dvořák and Postle [1] and was introduced as an extension of list-colorings of (undirected) graphs. It transforms the problem of finding a list-coloring of a given graph G with a list-assignment L to finding an independent transversal in an auxiliary graph with vertex set $\{(v, c) \mid v \in V(G), c \in L(v)\}$. In this talk, we extend the definition of DP-colorings to digraphs using the approach from Neumann-Lara where a coloring of a digraph is a coloring of the vertices such that the digraph does not contain any monochromatic directed cycle. Furthermore, we present a Brooks' type theorem for the DP-chromatic number, which extends various results on the (list-)chromatic number of digraphs.

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Minimum shortest path cover

Gabriel Semanišin

(joint work with Iztok Peterin)

Let G be a graph and u, v be vertices of G. A shortest path path P between vertices u and v is said to be *extendible* if there exist vertices x, y in G such that $\{u, v\} \neq \{x, y\}$ and P is a subpath of a shortest path between x and y.

Let $\mathcal{P}$ be the set of all shortest paths of a graph G that are not extendible. A subset $S \subseteq V(G)$ is called *minimum shortest path cover of* G if it has non-empty intersection with all paths belonging to $\mathcal{P}$. The cardinality of the smallest minimum shortest path cover in a graph G is denoted $\xi(G)$.

The new concept provides a generalisation of the minimum vertex cover and is motivated by a study of structural properties of graphs.

We investigate elementary properties of the introduced invariant, provide some lower and upper bounds and exact values for some classes of graphs.

Aharoni conjecture with more edges of each color

Aneta Šťastná

(joint work with Petra Pelikanová and Sophie Spirkl)

A directed graph is *simple*, if it does not contain any loops, parallel edges or digons (directed cycles of length two). A cycle in edge-colored graph is *rainbow* if all edges of the cycle have distinct colors.

Following conjecture by Caccetta and Häggkvist introduces a relation between outdegree of each vertex and presence of a short directed cycle:

Conjecture ([2]). *Every simple n-vertex directed graph with minimum outdegree at least k has a directed cycle with length at most $\lceil \frac{n}{k} \rceil$.*

This conjecture has been generalized by Aharoni for unoriented graphs and edge coloring:

Conjecture ([1]). *Let G be a simple n-vertex graph and c be a coloring of $E(G)$ with n colors, where each color class has size at least k. Then G contains a rainbow cycle of length at most $\lceil \frac{n}{k} \rceil$.*

We have shown that there exists a rainbow cycle of length $\lceil \frac{n}{k} \rceil$ if G contains at least $f(k) \in \mathcal{O}(k^2)$ edges of each color, using some partial results for these conjectures.

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Measures of edge-uncolorability of cubic graphs

Eckhard Steffen

There are many hard conjectures in graph theory, like Tutte's 5-flow conjecture, which would be true in general if they would be true for cubic graphs. Since most of them are trivially true for 3-edge-colorable cubic graphs, cubic graphs which are not 3-edge-colorable, often called snarks, play a key role in this context. In the talk we survey parameters measuring how far apart a snark is from being 3-edge-colorable. Besides getting new insight into the structure of snarks, we show that such measures are used to prove partial results with respect to some conjectures. The talk is based on the collection of results on this topic given in [1].

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Even dicuts and cut minors

Raphael Steiner

(joint work with Karl Heuer and Sebastian Wiederrecht)

An important graph class are the *Pfaffian graphs*, which admit a Pfaffian orientation [4]. The latter can be used to count the number of perfect matchings in Pfaffian graphs efficiently. While in general, the complexity of the recognition problem is open, a polynomial-time algorithm exists for bipartite graphs [2, 1]. The recognition of bipartite Pfaffian graphs is equivalent to the recognition of so-called *non-even digraphs*. These are the digraphs whose arcs can be $0, 1$-weighted such that every directed cycle has odd weight. This talk deals with a dual problem, namely, we consider *odd dijoins* in digraphs, which are arc sets intersecting every minimal dicut in an odd number of elements. We give a precise characterisation of digraphs admitting an odd dijoin in terms of certain forbidden minors. This result is analogous to a corresponding result for non-even digraphs by Seymour and Thomassen [3]. We show that the problem of testing for and finding an odd dijoin is polynomial-time equivalent to testing whether a given digraph contains a minimal dicut of even size. We present polynomial algorithms for special cases of the latter problem.

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Non-1-planarity of lexicographic products of graphs

Yusuke Suzuki

(joint work with Naoki Matsumoto)

We show the non-1-planarity of the lexicographic product of a theta graph and K_2. This result completes the proof of the conjecture, posed in [1], which states that a graph $G \circ K_2$ is 1-planar if and only if G has no edge belonging to two cycles.

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Hamiltonicity of lexicographic product

Jakub Teska

(joint work with Jan Ekstein)

The lexicographic product $G[H]$ of two graphs G and H is obtained from G by replacing each vertex with a copy of H and adding all edges between any pair of copies corresponding to adjacent vertices of G. In [2] Teichert and in [1] Kriessell found a sufficient conditions for Hamiltonicity of lexicographic product $P[H]$ of a path P and a graph H. We improved these these result by finding a full characterization of hamiltonian and traceable graphs $P[H]$ of lexicographic product of a path P and a graph H. Moreover, we proved even more general results. We found the already mentioned characterizations of graph $G[H_1, H_2, ..., H_n]$, which can be viewed as a generalization of lexicographic product.

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Strong edge coloring of graph products

Zsolt Tuza

(joint work with Suresh Dara, Suchismita Mishra, and Narayanan Narayanan)

As introduced some years before the first C&C Workshop, the strong chromatic index of a graph is the minimum number of colors in an edge coloring such that each color class is an induced matching. We prove lower and upper bounds and tight results for some product graphs. These include a subclass of Cayley graphs, and Cartesian products of two graphs where one factor is a tree and the other factor is a tree or a cycle.

Colouring non-even digraphs

Sebastian Wiederrecht

(joint work with Marcelo Garlet Millani and Raphael Steiner)

A colouring of a digraph as defined by Neumann-Lara in 1982 is a vertex-colouring such that no monochromatic directed cycles exist. The minimal number of colours required for such a colouring of a loopless digraph is defined to be its *dichromatic number*. This quantity has been widely studied in the last decades and can be considered as a natural directed analogue of the chromatic number of a graph. A digraph D is called *even* if for every 0-1-weighting of the edges it contains a directed cycle of even total weight. We show that every non-even digraph has dichromatic number at most 2 and an optimal colouring can be found in polynomial time. We strengthen a previously known NP-hardness result by showing that deciding whether a directed graph is 2-colourable remains NP-hard even if it contains a feedback vertex set of bounded size.

On directed versions of the 1-2-3 conjecture and the 1-2 conjecture

Mariusz Woźniak

Let $G = (V, E)$ be a graph. Given an integer k, a k-coloring (labeling) of G is a function $f : E \to \{1, 2, \ldots, k\}$. The coloring f can be represented by substituting each edge e of G by a multiedge with multiplicity $f(e)$. The degree of x in the respective multigraph equals the sum of labels around a vertex x. The 1-2-3 Conjecture says that for graphs without isolated edges there exists a 3-coloring f such the the corresponding multigraph is locally irregular *i.e.* for each edge xy of G we have $\sigma(x) \neq \sigma(y)$ where $\sigma(x) = \sum_{e \ni x} f(e)$. The 1-2 Conjecture refers to the case when we also color the vertices.

During the talk we shall look at the directed versions of these problems. We will discuss the results contained in the papers listed in the references below.

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Components condition and factors in graphs

Qinglin Roger Yu

Tutte's famous 1-factor theorem (1947) gave a characterization of perfect matchings in terms of odd components, namely, a graph G contains 1-factors if and only if $o(G - S) \leq |S|$ for any subset S of $V(G)$, where $o(G - S)$ is the number of odd components in $G - S$. A similar theorem using the number of isolated vertices gave a characterization of fractional perfect matching. So the numbers of odd components and isolated vertices can be good descriptions of specified spanning subgraphs. This observation inspires an interesting question: what kind of spanning subgraphs can be characterized by the condition $o(G - S) \leq \alpha|S|$ (or $o(G - S) \leq \beta|S|$)? In this talk, we survey the recent developments in this direction. The discussion on toughness as a type of component condition (i.e., $\omega(G - S) \leq \gamma|S|$), in particular the results related to cycle and coloring, are also included.

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