33rd Workshop

Cycles and Colourings

Nový Smokovec, August 31 – September 5, 2025

Book of Abstracts

https://candc.upjs.sk

Abstracts of the 33rd Workshop Cycles and Colourings
August 31 – September 5, 2025, Nový Smokovec, Slovakia

Editor: Igor Fabrici

Dear Participant,

welcome to the 33rd Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the 29th one in Košice (2021), and the 32nd one in Poprad (2024), the remaining twenty nine workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010, Nový Smokovec 2011–2019 and 2022–2023).

The workshop is organized by (current and former members of) the graph theory group of Institute of Mathematics, P.J. Šafárik University in Košice. It was created as a joint project and the first 30 series of C&C workshops were organized by graph theory groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks, the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Ilkyoo Choi	Hankuk University of Foreign Studies, Yongin-si, Republic of Korea
Hanna Furmańczyk	University of Gdańsk, Gdańsk, Poland
Penny E. Haxell	University of Waterloo, Waterloo, Canada
Balázs Keszegh	Alfréd Rényi Institute of Mathematics, Budapest, Hungary
Torsten Mütze	University of Kassel, Kassel, Germany
Robert Šámal	Charles University, Prague, Czech Republic

Have a pleasant and successful meeting.

Organising Committee:

Igor Fabrici
František Kardoš
Mária Maceková
Tomáš Madaras
Alfréd Onderko
Roman Soták

Programme

Sunday	
16:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday		
07:00 - 09:00	Breakfast	
09:00 - 09:50	**Haxell**	Graphs with high chromatic number
09:55 - 10:15	**Bujtás**	S-packing chromatic critical graphs
10:20 - 10:40	**Bešter Štorgel**	On tree-chromatic number and related notions
10:45 - 11:15	Coffee break	
11:15 - 11:35	**Jendroľ**	On a 3-coloring of plane graphs
11:40 - 12:00	**Masařík**	Proper rainbow saturation numbers for cycles
12:05 - 12:25	**Problem session**	
12:30 - 14:00	Lunch	
15:00 - 15:50	**Keszegh**	Coloring geometric hypergraphs
15:55 - 16:15	**Blázsik**	Small diameter covers
16:20 - 16:50	Coffee break	
16:50 - 17:10	**Mazák**	Circular chromatic index of graphs with $\Delta \in \{4, 5, 6\}$
17:15 - 17:35	**Renders**	HIST-critical graphs and Malkevitch's conjecture
17:40 - 18:00	**Onderko**	Totally locally irregular colorings of graphs
19:00 -	Welcome party	

Tuesday			
07:00 - 09:00		Breakfast	
09:00 - 09:50		**Mütze**	Counterexamples to two conjectures on Venn diagrams
09:55 - 10:15		**Škoviera**	Are there any permutation snarks of order 6 (mod 8)?
10:20 - 10:40		**Máčajová**	Rich nowhere-zero flows
10:45 - 11:15		Coffee break	
11:15 - 11:35		**Dvořák**	A strengthening of Alon-Tarsi method
11:40 - 12:00		**Przybyło**	On colourings and Alon-Tarsi number of hypergraphs
12:05 - 12:25		**Problem & Photo session**	
12:30 - 14:00		Lunch	
15:00 - 15:50		**Šámal**	Random embeddings of graphs
15:55 - 16:15		**Mikšaník**	On a refinement of the list chromatic index
16:20 - 16:50		Coffee break	
16:50 - 17:10	A	**Švecová**	Defective vertex coloring of toroidal graphs
	B	**Kamyczura**	Majority additive coloring
17:15 - 17:35	A	**Matok**	(Mostly) total colouring of (sub)cubic Halin graphs
	B	**Pękała**	List extensions of majority edge colourings
17:40 - 18:00	A	**Van den Eede**	Computational methods for finding bi-regular cages
	B	**Dettlaff**	Common independence in graphs
18:05 - 18:25	A	**Knor**	On the number of paths in a graph
	B		
18:30 - 20:00		Dinner	

Wednesday	
06:30 - 09:00	Breakfast
07:00 - 17:30	Trip
13:00 - 15:00	Lunch
18:00 - 20:00	Dinner

Thursday		
07:00 - 09:00	Breakfast	
09:00 - 09:50	**Furmańczyk**	b-coloring of regular graphs
09:55 - 10:15	**Tuza**	Nordhaus–Gaddum-type theorems for many subgraphs: Maximum average degree and more
10:20 - 10:40	**Barát**	Rainbow connecting coloring of Dirac graphs
10:45 - 11:15	Coffee break	
11:15 - 11:35	**Schiermeyer**	3-colourability, diamonds and butterflies
11:40 - 12:00	**Kalinowski**	The distinguishing chromatic number in hereditary graph classes
12:05 - 12:25	**Masaříková**	Constricting the computational complexity gap of the 4-coloring problem in (P_t, C_3)-free graphs
12:30 - 14:00	Lunch	
15:00 - 15:50	**Choi**	An improved lower bound on the number of edges in list critical graphs via DP coloring
15:55 - 16:15	**Provoost**	On a conjecture of Faudree and Schelp
16:20 - 16:50	Coffee break	
16:50 - 17:10	**Mattiolo**	Sets of r-graphs that color all r-graphs
17:15 - 17:35	**Rajník**	Generation of uniquely 3-edge-colourable graphs
17:40 - 18:00	**Ulyanov**	Oriented Berge–Fulkerson conjecture and unit vector flows on S^2
19:00 -	Farewell party	

Friday		
07:00 - 09:00	Breakfast	
09:00 - 09:20	**Cichacz**	On the conjecture of group antimagic trees
09:25 - 09:45	**Hedžet**	Induced matching vs edge open packing: trees and product graphs
09:50 - 10:10	**Samadi**	Independent mutual-visibility coloring
10:15 - 10:35	**Grzelec**	On weak homogeneous colorings
10:40 - 11:10	Coffee break	
11:10 - 11:30	**Oguagbaka**	Edge-uncoverability of cubic graphs
11:35 - 11:55	**Řehulka**	Cyclic edge-connectivity of junctions of multipoles
11:45 - 14:00	Lunch	

Rainbow connecting coloring of Dirac graphs

János Barát

(joint work with Simona Boyadzhiyska)

An edge-colored graph G is *rainbow connected* if every two vertices of G are connected by a rainbow path (that is a path with all its edges colored with different colors) [2, 3]. The minimum number of colors required so that the colored graph is rainbow connected, is called the rainbow connection number, and it is denoted by $\mathrm{rc}(G)$. At last year's meeting, Geňa Hahn advertised the following [1].

Problem Does there exist a constant $c \geq 0$ such that if G is a non-complete graph of order n with $\delta(G) \geq \frac{n}{2} + c$, then $\mathrm{rc}(G) = 2$? In particular, does $\delta(G) \geq \frac{n}{2}$ imply that $\mathrm{rc}(G) = 2$?

We concentrate on $rc(G) = 2$, and call the corresponding coloring an rc2-coloring. Gimbel noticed that the complete bipartite graph $K_{\frac{n}{2},\frac{n}{2}}$ admits a simple rc2-coloring, where one color class is a perfect matching. Several other examples showed that possibly one color class is a forest or has very few edges. We posed and answered the following natural questions:

- Does every graph G that admits an rc2-coloring have one in which one color class has at most $n/2$ edges?
- What is the smallest function $f(n)$ such that every graph G that admits an rc2-coloring have one in which one color class has at most $f(n) \cdot n$ edges? Can $f(n)$ be a constant?
- Does every graph G that admits an rc2-coloring have one in which one color class is a forest?
- Is it true that every graph in which every pair of non-adjacent vertices has at least 2 common neighbors has an rc2-coloring?
- Is it true that every graph G in which every pair u, v of non-adjacent vertices satisfies $\deg(u) + \deg(v) \geq n - 1$ has an rc2-coloring?

References

[1] J. Barát, S. Cambie, G. Hahn, D. Mattiolo, A. Onderko, I. Schiermeyer, Zs. Tuza, Open problems of the 32nd Workshop on Cycles and Colourings, arXiv: 2411.10046 (2024).

[2] Y. Caro, A. Lev, Y. Roddity, Z. Tuza, R. Yuster, On rainbow connection, Electron. J. Comb. 15 (2008), #R57.

[3] G. Chartrand, Highly irregular, In: R. Gera, T.W. Heynes, S.T. Hedetniemi (eds.), Graph Theory: Favorite Conjectures and Open Problems-1, Springer, 2016, 1–16.

[4] J. Gimbel, A conjecture on rainbow connectivity, ITI Series, Charles University, 687 (2024).

On tree-chromatic number and related notions

Kenny Bešter Štorgel

The notion of tree-chromatic number was introduced in 2016 by Seymour [2] as a chromatic variant of treewidth, where the cost of a bag in a tree decomposition is measured by its chromatic number rather than its size. Formally, a graph G has tree-chromatic number at most k if it admits a tree decomposition $\mathcal{T}$ such that every bag of $\mathcal{T}$ induces a subgraph with chromatic number at most k. Since its introduction, the tree-chromatic number has not yet been examined extensively.

In recent years another similar concept gained the attention of many researchers. Namely, the tree-independence number, introduced independently by Yolov [3] in 2018 and Dallard, Milanič, and Štorgel [1] in 2024. The tree-independence number is an independence variant of treewidth. Formally, a graph G has tree-independence number at most k if it admits a tree decomposition $\mathcal{T}$ such that every bag of $\mathcal{T}$ induces a subgraph with independence number at most k.

In [1] the authors asked if treewidth of a graph (plus one) can always be upper bounded by the product of its tree-chromatic number and its tree-independence number, supporting the question by showing that the inequality holds in the class of perfect graphs. Although this question was recently answered in the negative by Krause in his Master's Thesis (in 2024), a question remains of identifying for which graphs the inequality holds. In this talk, we will give an overview of known properties of the tree-chromatic and establish its relation with the tree-independence number and some other invariants.

References

[1] C. Dallard, M. Milanič, K. Štorgel, Treewidth versus clique number. II. Tree-independence number, J. Combin. Theory Ser. B 164 (2024), 404–442.

[2] P. Seymour, Tree-chromatic number, J. Combin. Theory Ser. B 116 (2016), 229–237.

[3] N. Yolov, Minor-matching hypertree width, Proc. SODA 2018, 219–233.

Small diameter covers

Zoltán L. Blázsik

(joint work with Recep Altar Çiçeksiz, Balázs Keszegh, and Laurentiu Ploscaru)

In the last 5 years, a lot of interest was shown in bounded diameter variations of Ryser's conjecture. The famous conjecture asserts the existence of a vertex cover of $(r-1)\alpha$ monochromatic connected components for every graph $G(V,E)$ with independence number $\alpha(G) = \alpha$ and for every r-edge-coloring of G. Milićević initiated the question whether the diameters of the covering components can be bounded.

Very recently Gyárfás and Sárközy [1] proved for $\alpha = 2$ there always exist 2 monochromatic connected components with diameter at most 4 covering the vertex set for any 2-edge-coloring. They also proposed another version, namely if the diameter of the monochromatic connected components are bounded from above by d, for some fixed d, then how many of them is needed to cover the vertex set for any 2-edge-coloring? They proved that $\lfloor 3\alpha/2 \rfloor$ many components are always enough for $d = 4$.

On the other hand, DeBiasio, Girão, Haxell and Stein [2] proved that for any 2-edge-coloring of $G(V, E)$ there always exist α many monochromatic connected components covering V such that the diameters of these components are at most $8\alpha^2 + 12\alpha + 6$.

In this talk, I would like to connect these two results by showing that if it is allowed to use monochromatic connected components of diameter at most $d(\alpha)$, then $\alpha + \frac{3\alpha}{2\sqrt{d(\alpha)}}$ many of them is enough to cover V for any 2-edge-coloring.

References

[1] A. Gyárfás, G.N. Sárközy, Bounded diameter variations of Ryser's conjecture, arXiv:2505.02564 (2025).

[2] L. DeBiasio, A. Girão, P. Haxell, M. Stein, A bounded diameter strengthening of Kőnig's theorem, SIAM J. Discrete Math. 39 (2025), 24M1701022.

S-packing chromatic critical graphs

Csilla Bujtás

(joint work with Gülnaz Boruzanlı Ekinci, Didem Gözüpek, and Sandi Klavžar)

For a non-decreasing sequence of positive integers $S = (s_1, s_2, \ldots)$, the S-packing chromatic number of a graph G is denoted by $\chi_S(G)$. This talk is based on [1], where χ_S-critical graphs are introduced as the graphs G such that $\chi_S(H) < \chi_S(G)$ for each proper subgraph H of G. Several families of χ_S-critical graphs are constructed, and 2- and 3-colorable χ_S-critical graphs are presented for all packing sequences S, while 4-colorable χ_S-critical graphs are found for most of S. Cycles which are χ_S-critical are characterized under different conditions. It is proved that for every graph G and every edge $e \in E(G)$, the inequality $\chi_S(G - e) \geq \chi_S(G)/2$ holds. Moreover, in several important cases, this bound can be improved to $\chi_S(G - e) \geq (\chi_S(G) + 1)/2$. The sharpness of the bounds is also discussed.

References

[1] G. Boruzanlı Ekinci, Cs. Bujtás, D. Gözüpek, S. Klavžar, S-packing chromatic critical graphs, arXiv:2505.17369 (2025).

An improved lower bound on the number of edges in list critical graphs via DP coloring

Ilkyoo Choi

(joint work with Peter Bradshaw, Alexandr V. Kostochka, and Jingwei Xu)

A graph G is (list, DP) k-critical if the (list, DP) chromatic number is k but for every proper subgraph G' of G, the (list, DP) chromatic number of G' is less than k. For $k \geq 4$, we show a bound on the minimum number of edges in a DP k-critical graph, and our bound is the first bound that is asymptotically better than the corresponding bound for proper k-critical graphs by Gallai from 1963. Our result also improves the best bound on the list chromatic number.

On the conjecture of group antimagic trees

Sylwia Cichacz

Kaplan, Lev and Roditty considered the following generalization of the concept of an antimagic graph.

Let $(\Gamma, +)$ be an Abelian group (not necessarily finite) and A be a finite subset of $\Gamma^* = \Gamma \setminus \{0\}$ with $|A| = |E(G)|$. An A-labeling of $G = (V, E)$ is a one-to-one mapping $f\colon E(G) \to A$. The weight of every vertex is calculated as the sum (taken in Γ) of the labels of incident edges. We say that G is *A-antimagic* if all the weights differ.

Let $\Gamma^* = \Gamma \setminus \{0\}$. Kaplan, Lev, and Roditty conjectured that a tree with $|\Gamma|$ vertices is Γ^*-antimagic if and only if the number of involutions in Γ is not one. In this talk, we show that the conjecture is not true. Moreover, we prove that paths $P_{|\Gamma|}$ are Γ-antimagic if and only if $|\Gamma| \not\equiv 2 \pmod 4$.

Common independence in graphs

Magda Dettlaff

(joint work with Magdalena Lemańska and Jerzy Topp)

The cardinality of a largest independent set of G, denoted by $\alpha(G)$, is called the independence number of G. The independent domination number $i(G)$ of a graph G is the cardinality of a smallest independent dominating set of G. The graphs G for which we have $i(G) = \alpha(G)$ are known as well-covered graphs. In other words, a graph G is well-covered if every maximal independent set of vertices in G is also a largest independent set. The concept of well-covered graphs was introduced by Plummer [2] and extensively studied in many papers.

Now, between the integers $i(G)$ and $\alpha(G)$, we insert another integer concerning the existence and cardinality of independent sets in G. Formally, in [1] we introduce the concept of the *common independence number* of a graph G, denoted by $\alpha_c(G)$, as the greatest integer r such that every vertex of G belongs to some independent subset X of V_G with $|X| \geq r$. Thus, the common independence number of a graph G refers to numbers of mutually independent vertices of G and it emphasizes the notion of the individual independence of a vertex of G from other vertices of G. The common independence number $\alpha_c(G)$ of G is the limit of symmetry in G with respect to the fact that each vertex of G belongs to an independent set of cardinality $\alpha_c(G)$ in G, and there are vertices in G that do not belong to any larger independent set in G.

It follows immediately from the above definitions that $i(G) \leq \alpha_c(G) \leq \alpha(G)$ for any graph G. We focus on the family of graphs G with equalities: $i(G) = \alpha_c(G)$ or $\alpha_c(G) = \alpha(G)$, in particular in [1] we characterize the trees T for which $i(T) = \alpha_c(T)$, and the block graphs G for which $\alpha_c(G) = \alpha(G)$. Obviously the family well-covered graphs is a proper subfamily of both considered families.

References

[1] M. Dettlaff, M. Lemańska, J. Topp, Common independence in graphs, Symmetry 13 (2021), 1411.

[2] M.D. Plummer, Some covering concepts in graphs, J. Combin. Theory 8 (1970), 91–98.

A strengthening of Alon-Tarsi method

Zdeněk Dvořák

A well-known result of Alon and Tarsi states that if the coefficient of the monomial $\prod_{v\in V(G)} x_v^{d_v}$ in the graph polynomial of a graph G is non-zero, then G is L-colorable for any list assignment L such that $|L(v)| > d_v$ for every $v \in V(G)$. This provided an algebraic way to show that a graph is list-colorable. A downside is that if the relevant coefficients are zero, this does not tell us anything about the list colorability of the graph. In my talk, I describe a strengthening of this result which uses further coefficients of the graph polynomial to restrict the list assignments L for which G is not L-colorable, potentially showing the L-colorability even in the cases where the original method fails.

b-coloring of regular graphs

Hanna Furmańczyk

(joint work with Magda Dettlaff, Iztok Peterin,
Adriana Roux, and Radosław Ziemann)

Let G be a graph and $c : V(G) \to \{1, \ldots, k\}$ a proper k-coloring of G, i.e., $c(u) \neq c(v)$ for every edge uv from G. A proper k-coloring is a b-coloring if there exists a vertex in every color class that contains all the colors in its closed neighborhood. The maximum number of colors k admitting b-coloring of G is the b-chromatic number $\chi_b(G)$.

We present two separate approaches to the conjecture posed by Blidia et. al [1] that $\chi_b(G) = d + 1$ for every d-regular graph of girth at least five except the Petersen graph.

The main conjecture for d-regular graphs was posed by Blidia et al. [1].

Conjecture 1 ([1]) *Every d-regular graph of girth at least 5, different than Petersen graph, has a b-coloring with $d + 1$ colors.*

We are attacking the above mentioned conjecture by two different approaches. First we deal with the case when there exists a vertex x that does not belong to too many six cycles which have all vertices on a distance at most two from x. In particular, we prove Conjecture 1 when there is such a vertex with at most five mentioned six cycles.

Theorem 1 ([2]) *Let G be a d-regular graph, $d \geq 7$, with $g(G) = 5$. If there exists a vertex in G that belongs to no cycle C_6, then $\chi_b(G) = d + 1$.*

Theorem 2 ([2]) *Let G be a d-regular graph, $d \geq 7$, with $g(G) = 5$. If there exists a vertex x in G contained in at most 5 cycles C_6 in $G[N_2[x]]$, then $\chi_b(G) = d + 1$.*

The second approach assumes that all the neighbors of two neighbors of x are at distance at most two to x. With this we add a new brick to the confirmation of Conjecture 1.

Theorem 3 ([2]) *Let G be a d-regular graph, $d \geq 7$, of the girth 5. If there exists a vertex x in $V(G)$ with at least two bunches where all the neighbors of vertices in a bunch are in $N_2(x)$, then $\chi_b(G) = d + 1$.*

References

[1] M. Blidia, F. Maffray, Z. Zemir, On b-colorings in regular graphs, Discrete Appl. Math. 157 (2009), 1787–1793.

[2] M. Dettlaff, H. Furmańczyk, I. Peterin, A. Roux, R. Ziemann, A new approach to b-coloring of regular graphs, Appl. Math. Comput. 481 (2024), 128914.

On weak homogeneous colorings

Igor Grzelec

(joint work with Tomáš Madaras and Alfréd Onderko)

We say that vertex k-coloring of a graph G (not necessarily proper) is weak l-homogeneous if the number of colors in the neighborhood of each vertex of G equals l. After short introduction about proper vertex k-colorings of graphs that are l-homogeneous introduced by Tomáš Madaras and Mária Šurimová in [1] we present weak l-homogeneous colorings and other closely related problems that provide results on them, among others concept of total domatic number and R6-coloring. Then we provide Integer Linear Programming formulation of the problem of finding weak l-homogeneous coloring of a graph. We also present the computational complexity of the problem of deciding whether given graph has l-homogeneous k-coloring. We also introduce some results about 2-homogeneous colorings of different families of sparse graphs. At the end we show some examples of graphs that do not have l-homogeneous colorings for any $l \geq 2$.

References

[1] T. Madaras, M. Šurimová, Homogeneous colourings of graphs, Math. Bohem. 148 (2023), 105–115.

Graphs with high chromatic number

Penny E. Haxell

The classical theorem of Brooks tells us that if a graph G has no colouring with its maximum degree $\Delta \geq 3$ colours, then it contains a clique with $\Delta+1$ vertices. Does a similar phenomenon occur when the chromatic number is slightly smaller than Δ? Even the next case is unknown: in 1977, Borodin and Kostochka famously conjectured that if $\Delta \geq 9$ and G has no $(\Delta - 1)$-colouring then it contains a $\Delta(G)$-clique.

We discuss various results and questions around this conjecture, as well as a related conjecture of Reed that seeks to quantify more generally how large a clique a graph should contain if its chromatic number is close to its maximum degree.

Induced matching vs edge open packing: trees and product graphs

Jaka Hedžet

(joint work with Boštjan Brešar, Tanja Dravec, and Babak Samadi)

Given a graph G, the maximum size of an induced subgraph of G each component of which is a star is called the edge open packing number, $\rho_e^o(G)$, of G. Similarly, the maximum size of an induced subgraph of G each component of which is the star $K_{1,1}$ is the induced matching number, $\nu_I(G)$, of G. While the inequality $\rho_e^o(G) \geq \nu_I(G)$ clearly holds for all graphs G, we provide a structural characterization of those trees that attain the equality. We prove that the induced matching number of the lexicographic product $G \circ H$ of arbitrary two graphs G and H equals $\alpha(G)\nu_I(H)$. By similar techniques, we prove sharp lower and upper bounds on the edge open packing number of the lexicographic product of graphs, which in particular lead to NP-hardness results in triangular graphs for both invariants studied in this paper. For the direct product $G \times H$ of two graphs we provide lower bounds on $\nu_I(G \times H)$ and $\rho_e^o(G \times H)$, both of which are widely sharp. We also present sharp lower bounds for both invariants in the Cartesian and the strong product of two graphs. Finally, we consider the edge open packing number in hypercubes establishing the exact values of $\rho_e^o(Q_n)$ when n is a power of 2, and present a closed formula for the induced matching number of the rooted product of arbitrary two graphs over an arbitrary root vertex.

On a 3-coloring of plane graphs

Stanislav Jendrol'

A facial path in a plane graph G is a subpath of the boundary walk of a face of G. The Four Color Theorem states that every plane graph contains a proper vertex 4-coloring in which each monochromatic path consists of exactly one vertex. Czap, Fabrici, and Jendrol' in 2021 conjectured that every plane graph G admits an improper vertex 3-coloring in which every monochromatic facial path in G has at most two vertices. In this talk we prove this conjecture. Our result is optimal.

References

[1] J. Czap, I. Fabrici, S. Jendrol', Colorings of plane graphs without long monochromatic facial paths, Discuss. Math. Graph Theory 41 (2021), 801–808.

The distinguishing chromatic number in hereditary graph classes

Rafał Kalinowski

(joint work with Christoph Brause, Monika Pilśniak, and Ingo Schiermeyer)

The distinguishing chromatic number of a graph G, denoted $\chi_D(G)$, is the minimum number of colours in a proper vertex colouring of G that is preserved by the identity automorphism only. This concept was introduced by Collins and Trenk, who proved that $\chi_D(G) \leq 2\Delta(G)$ for any connected graph G, and the equality holds only for complete balanced bipartite graphs $K_{p,p}$ and for C_6. We show that the upper bound on $\chi_D(G)$ can be substantially reduced if we forbid some small graphs as induced subgraphs of G, that is, we study the distinguishing chromatic number in some hereditary graph classes.

Majority additive coloring

Mateusz Kamyczura

Majority additive coloring is a type of coloring where each vertex is assigned a number, and the sum of its neighbors' numbers, called the neighbor sum, is then computed. For the coloring to be valid, in the neighborhood of each vertex, at most half of its neighbors can share the same neighbor sum. Therefore, majority additive coloring is a combination of two known problems: additive coloring and majority coloring. The majority additive chromatic number, denoted by $\chi_{mac}(G)$, is the smallest number of colors required to achieve a majority additive coloring of G. We present several results regarding χ_{mac} for different types of graphs. For complete graphs and cycles, we have determined the exact value of the parameter, while for trees, we have found a tight upper bound. The main result of this work shows that for graphs with girth greater than 5, a sufficiently large maximum degree, and a minimum degree close to the maximum degree, it is sufficient to use only the numbers 1 and 2.

Coloring geometric hypergraphs

Balázs Keszegh

A proper coloring of the vertices of a hypergraph is a coloring such that every hyperedge of size at least 2 is not monochromatic.

Given a set of points S and a family of regions $\mathcal{F}$ in the plane, we can define the *geometric hypergraph* $H(S, \mathcal{F})$, whose vertex set is S and its hyperedges are those subsets of S that can be obtained as $S \cap F$ for some $F \in \mathcal{F}$.

Pach asked in 1980 if an m-fold covering of the plane with translates of a convex region can be always decomposed into two 1-fold coverings for large enough m. This motivated the following problem.

Given a geometric hypergraph H defined by some type of family of regions, what is the smallest m such that the hypergraph H_m formed by the hyperedges of H of size at least m is always two-colorable? If $\mathcal{F}$ is a finite subfamily of a given nice family then can we bound m by a constant that depends only on the nice family and not on the specific subfamily? The same problems for the dual of a geometric hypergraph are similarly relevant.

We survey the most important results and recent directions of this area, concentrating on results when $\mathcal{F}$ is a family of (pseudo)disks, (pseudo)halfplanes, unit disks, translates of a convex region, homothets of a convex region.

For a collection of results of this area see the interactive database maintained by the speaker and Dömötör Pálvölgyi (https://coge.elte.hu/cogezoo.html).

On the number of paths in a graph

Martin Knor

(joint work with Jelena Sedlar, Riste Škrekovski, and Yu Yang)

We count the number of (not necessarily induced) paths in a graph. Extremal values of this invariant are determined for connected graphs. Then we consider two classes of graphs.

In the class of cacti on n vertices with k cycles, $C(n, k)$, we show that the graph minimizing the Wiener index is among the graphs with the minimum number of paths. But the graph from $C(n, k)$ which maximizes the number of paths is different from the graph which maximizes the Wiener index. Here we recall that in $C(n, k)$, the graph which maximizes the Wiener index is exactly the graph which minimizes the subtree number and vice versa.

The second class of graphs is the sequence of cycles, where consecutive cycles share exactly one edge and the maximum degree is 3. This class covers the ladder graph as well as hexagonal chains. We calculate the exact formula for the number of paths in these graphs and determine the extremal graphs if the sizes of cycles are given.

Rich nowhere-zero flows

Edita Máčajová

(joint work with Karolína Pisoňová)

A graph admits a nowhere-zero k-flow if its edges can be oriented and assigned values the set $\{1, 2, ..., k-1\}$ in such a way that, at every vertex, the sum of the incoming values equals the sum of the outgoing values. The concept of a nowhere-zero flow is one of the most important concepts in graph theory and has been studied for more than half a century.

In this talk, we introduce – for cubic graphs – the notion of a *rich* nowhere-zero k-flow: one where the three values at every vertex are pairwise distinct. We conjecture that a cubic graph admits a nowhere-zero k-flow if and only if it admits a rich nowhere-zero $(k+1)$-flow. We prove the this conjecture for $k = 3$ and $k = 4$, that is, for bipartite and 3-edge-colourable cubic graphs, respectively. Furthermore, we show that every bridgeless cubic graph admits a rich nowhere-zero 11-flow and conjecture 6 in place of 11 will do.

Proper rainbow saturation numbers for cycles

Tomáš Masařík

(joint work with Anastasia Halfpap and Bernard Lidický)

We say that an edge-coloring of a graph G is *proper* if every pair of incident edges receive distinct colors, and is *rainbow* if no two edges of G receive the same color. Furthermore, given a fixed graph F, we say that G is *(proper) rainbow F-saturated* if G admits a proper edge-coloring which does not contain any rainbow subgraph isomorphic to F, but the addition of any edge to G makes such an edge-coloring impossible. The maximum number of edges in a (proper) rainbow F-saturated graph is the rainbow Turán number, whose study was initiated in 2007 by Keevash, Mubayi, Sudakov, and Verstraëte [2]. Recently, Bushaw, Johnston, and Rombach [1] introduced study of a corresponding saturation problem, asking for the *minimum* number of edges in a (proper) rainbow F-saturated graph. We term this minimum the *proper rainbow saturation number* of F, denoted $\mathrm{sat}^*(n, F)$. We asymptotically determine $\mathrm{sat}^*(n, C_4)$, answering a question of [1]. We also exhibit constructions which establish upper bounds for $\mathrm{sat}^*(n, C_5)$ and $\mathrm{sat}^*(n, C_6)$.

References

[1] N. Bushaw, D. Johnston, P. Rombach, Rainbow saturation, Graphs Combin. 38 (2022), #166.

[2] P. Keevash, D. Mubayi, B. Sudakov, J. Verstraëte, Rainbow Turán problems, Combin. Probab. Comput. 16 (2007), 109–126.

Constricting the computational complexity gap of the 4-coloring problem in (P_t, C_3)-free graphs

Jana Masaříková

(joint work with Justyna Jaworska, Bartłomiej Kielak, and Tomáš Masařík)

The k-coloring problem on hereditary graph classes has been a deeply-researched problem in the last decade. A hereditary graph class is characterized by a (possibly infinite) list of minimal forbidden induced subgraphs. We denote a graph $(H_1, H_2, \ldots)$-*free* if it does not contain any of $H_1, H_2, \ldots$ graphs as induced subgraphs. The complexity landscape of the problem remains unclear even when restricting to case $k = 4$ for classes defined by a few forbidden induced subgraphs. While the case of only one forbidden induced subgraph has been completely resolved recently, the complexity when two forbidden induced subgraphs are considered still has a couple of unknown cases. In particular, 4-coloring on (P_6, C_3)-free graphs is polynomial while it is NP-hard on (P_{22}, C_3)-free graphs.

We provide a construction showing NP-completeness of (P_t, C_3)-free graphs for $19 \leq t \leq 21$ and thus constricting the gap of cases whose complexity remains unknown. Our construction is partly based on computer search that allows us to analyze the structure of induced paths in parts of our gadgets.

(Mostly) total colouring of (sub)cubic Halin graphs

Matúš Matok

(joint work with František Kardoš)

A Halin graph is a planar graph consisting of a tree and an additional cycle connecting all the leaves in such manner that no two edges are crossing. We will be interested in (sub)cubic Halin graphs.

Total colouring of a graph is a mapping from the set of vertices and edges to a set of colours such that no two neighbouring objects receive the same colour. We will be (mostly) talking about total colouring, but we will also briefly mention two very related colourings, namely AVD-colouring and SND-colouring.

As there were only 3 known cubic Halin graphs with total chromatic index greater than 4, a natural open question was informally posed by Borut Lužar on whether the number of such graphs is finite.

We managed to prove that the set of cubic Halin graphs with total chromatic index greater than 4 is finite, containing only the cubic Halin graphs known before-hand. By a slight modification of our approach, we managed to establish similar results for total and AVD-colouring of subcubic Halin graphs. Lastly, we observed that the set of subcubic Halin graphs with SND-chromatic index greater than 4 is infinite and its precise characterisation is currently a work in progress.

Sets of r-graphs that color all r-graphs

Davide Mattiolo

(joint work with Yulai Ma, Eckhard Steffen, and Isaak H. Wolf)

An r-regular graph is an r-*graph* if every odd set of vertices is connected to its complement by at least r edges. Let G and H be r-graphs. An H-*coloring* of G is a mapping $f\colon E(G) \to E(H)$ such that each r adjacent edges of G are mapped to r adjacent edges of H. For every $r \geq 3$, let $\mathcal{H}_r$ be an inclusion-wise minimal set of connected r-graphs, such that for every connected r-graph G there is $H \in \mathcal{H}_r$ which colors G. The Petersen Coloring Conjecture states that $\mathcal{H}_3$ consists of the Petersen graph P. We show that if true, then this is a very exclusive situation. Our main result is that either $\mathcal{H}_3 = \{P\}$ or $\mathcal{H}_3$ is an infinite set and if $r \geq 4$, then $\mathcal{H}_r$ is an infinite set. In particular, for all $r \geq 3$, $\mathcal{H}_r$ is unique and we characterize it by showing that $G \in \mathcal{H}_r$ if and only if the only connected r-graph coloring G is G itself.

Circular chromatic index of graphs with $\Delta \in \{4, 5, 6\}$

Ján Mazák

(joint work with Filip Zrubák)

According to the Vizing theorem, circular chromatic index χ'_c of a simple graph with maximum degree Δ is a fraction from the interval $[\Delta, \Delta+1]$. Each fraction from the subinterval $[\Delta, \Delta + 1/6]$ is attained, yet it was conjectured that there is a subinterval $[\Delta + 1 - \delta, \Delta + 1]$ for some $\delta > 0$ such that no graph has χ'_c in this subinterval (it is proved for $\Delta \leq 3$). We provide computational results for $\Delta \in \{4, 5, 6\}$ and discuss possible relationships between δ and Δ, trying to make the above-given conjecture more precise (and refuting certain variants of it appearing in the literature).

Apart from Δ-regular graphs, we also explore graphs and multigraphs with maximum degree Δ. Some of the found graphs with specific values of χ'_c can be generalized into infinite classes. We also describe an infinite family of 4-connected 4-regular graphs with $\chi'_c = 4 + 1/2$.

On a refinement of the list chromatic index

David Mikšaník

(joint work with Ross Kang)

In our talk, graphs are allowed to have parallel edges but not loops. Furthermore, graphs with no parallel edges are called *simple*. The famous List Coloring Conjecture from the 1970s states that, for any graph G, the chromatic index $\chi'(G)$

of G equals the list chromatic index $\chi'_\ell(G)$ of G. The conjecture is known to hold asymptotically: Jeff Kahn in 1996 proved that $\chi'_\ell(G) = (1 + o(1))\chi'(G)$, where $o(1)$ is a term that goes to zero as $\Delta(G)$ goes to infinity [1]. Simpler proofs of this result are known for simple graphs (see e.g. [2] or [3, Chapter 14]).

We consider a refinement of the list chromatic index $\chi'_\ell(G)$, which has been introduced in the master's thesis of Lotte Paterek [4]. For a natural number $d \in \mathbb{N}$, we define $\chi'_{\ell,d}(G)$ to be the smallest $k \in \mathbb{N}$ such that G is L-edge-colorable for every edge-list assignment L with $|L(e)| \geq k$ for every $e \in E(G)$ and $d(v, \alpha) := |\{u \in N(v) \colon \alpha \in L(\{u, v\})\}| \leq d$ for every $v \in V(G)$ and every color α. Our main result states that, for any simple graph G, we have $\chi'_{\ell,d}(G) = (1+o(1))d$, where here $o(1)$ is a term that goes to zero as d goes to infinity. Since clearly $\chi'_\ell(G) = \chi'_{\ell,\Delta(G)}(G)$, this generalizes the result stated in the first paragraph for simple graphs.

We briefly discuss the proof of our main result. However, our main focus will be on presenting several new open problems related to this new graph coloring parameter.

References

[1] J. Kahn, Asymptotically good list-colorings, J. Combin. Theory Ser. A 73 (1996), 1–59.

[2] R. Häggkvist, J. Janssen, New bounds on the list-chromatic index of the complete graph and other simple graphs, Combin. Probab. Comput. 6 (1997), 295–313.

[3] M. Molloy, B. Reed, Graph colouring and the probabilistic method, Springer, 2002.

[4] L. Paterek, Perspectives on the List Colouring Conjecture, MSc in Mathematics thesis, University of Amsterdam (2025).

Counterexamples to two conjectures on Venn diagrams

Torsten Mütze

(joint work with Sofia Brenner, Linda Kleist, Christian Rieck, and Francesco Verciani)

In 1984, Winkler [1] conjectured that every simple Venn diagram with n curves can be extended to a simple Venn diagram with $n + 1$ curves. His conjecture is equivalent to the statement that the dual graph of any simple Venn diagram has a Hamilton cycle. In this work, we construct counterexamples to Winkler's conjecture for all $n \geq 6$; see Figure 1. As part of this proof, we computed all 3.430.404 simple Venn diagrams with $n = 6$ curves (even their number was not previously known), among which we found 72 counterexamples.

We also disprove another conjecture about the Hamiltonicity of the (primal) graph of a Venn diagram. Specifically, while working on Winkler's conjecture, Pruesse and Ruskey [2] proved that this graph has a Hamilton cycle for every simple Venn diagram with n curves, and conjectured that this also holds for non-simple diagrams. We construct counterexamples to this conjecture for all $n \geq 4$.

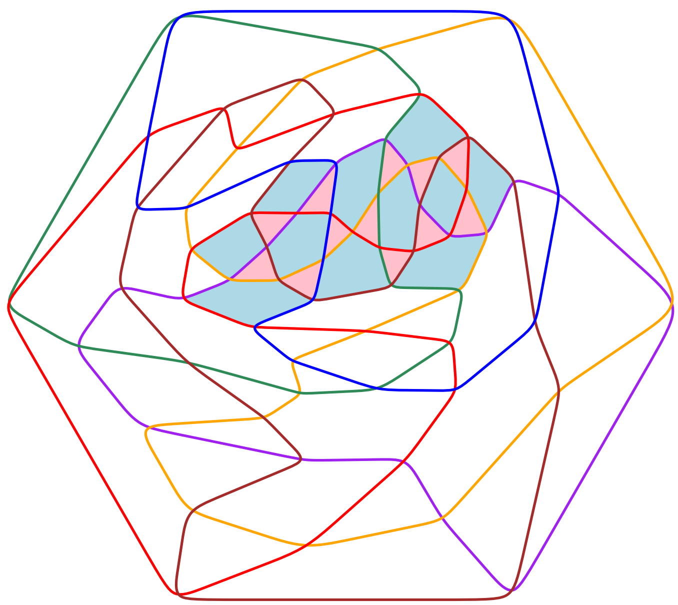


Figure 1: A counterexample to Winkler's conjecture for $n = 6$, namely a simple 6-Venn diagram that cannot be extended to a simple 7-Venn diagram by adding a suitable curve.

References

[1] P. Winkler, Venn diagrams: some observations and an open problem, Congr. Numer. 45 (1984), 267–274.

[2] G. Pruesse, F. Ruskey, All simple Venn diagrams are Hamiltonian, arXiv: 1504.06651 (2015).

Edge-uncoverability of cubic graphs

Makuochukwu Felix Oguagbaka

(joint work with Robert Lukoťka)

An extension of Petersen's Perfect Matching Theorem by Schönberger implies that every bridgeless cubic graph admits a set of perfect matchings covering all its edges. Berge conjectured that at most five perfect matchings suffice. Cubic graphs that require at least five perfect matchings to cover their edges are particularly interesting, as they could serve as counterexamples to famous conjectures, including the Cycle Double Cover Conjecture. On the other hand, cubic graphs whose edges can be covered by four perfect matchings are known to satisfy these conjectures.

At last year's Workshop Cycles and Colourings, we introduced several invariants to measure how far a cubic graph is from being coverable by four perfect matchings. One of the invariants is the *four perfect matching cover defect*, defined as the minimum number of edges of a cubic graph that cannot be covered by four perfect matchings. In this talk, we review existing results and present our recent findings on these invariants and their relation to well-known conjectures.

Totally locally irregular colorings of graphs

Alfréd Onderko

(joint work with Anna Flaszczyńska,
Aleksandra Gorzkowska, Igor Grzelec, and Mariusz Woźniak)

In 2015, Baudon et at. [1] defined a new problem connected to local irregularity in graphs: they asked whether for every graph G there is a total coloring $c\colon E(G) \cup V(G) \to \{1, 2\}$ such that for every edge uv of color $c(uv) = i$ the numbers of elements colored with i from the sets $\{u\} \cup \{uw\colon w \in N(u)\}$ and $\{v\} \cup \{vw\colon w \in N(v)\}$ are different. Such a coloring is called a totally locally irregular 2-coloring (TLIR 2-coloring in short). The motivation was the equivalence of the conjecture that every regular graph admits a TLIR 2-coloring and the 1-2 Conjecture [3] for regular graphs. Recently, the 1-2 Conjecture was proved by Deng and Qiu for regular graphs [2]. We utilize their construction and prove that every subcubic graph has a TLIR 2-coloring. Furthermore, we show that every cactus graph has a TLIR 2-coloring. These two classes are interesting when considering the local irregularity of graphs, as the only graphs that do not have a locally irregular edge coloring [1] (the original problem of locally irregular colorings) are special subcubic cacti. In addition to these results, we provide upper bounds on the minimum number of colors required for a TLIR coloring in the case of general graphs, using chromatic number and star arboricity. In particular, this yields relatively small upper bounds in the case of outerplanar and planar graphs, which improves the bounds implied by the results on locally irregular edge colorings of the graphs from these classes.

References

[1] O. Baudon, J. Bensmail, J. Przybyło, M. Woźniak, On decomposing regular graphs into locally irregular subgraphs, European J. Combin. 49 (2015), 90–104.

[2] K. Deng, H. Qiu, The 1-2 conjecture holds for regular graphs, J. Combin. Theory Ser. B 174 (2025), 207–213.

[3] J. Przybyło, M. Woźniak, On a 1, 2 conjecture, Discrete Math. Theor. Comput. Sci. 12 (2010), 101–108.

List extensions of majority edge colourings

Paweł Pękała

(joint work with Jakub Przybyło)

A *majority edge colouring* of a graph G is a colouring of the edges of G such that for each vertex v of G, at most half the edges incident with v have the same colour. More generally, for a natural number $k \geq 2$, a $1/k$-*majority edge-colouring* of a graph is a colouring of the edges of G such that for every colour c and every vertex v of G at most $1/k$ of the edges incident with v have the colour c. This notion was introduced in 2023 by Bock, Kalinowski, Pardey, Pilśniak, Rautenbach and Woźniak [1].

We investigate possible list extensions of generalised majority edge colourings. In particular, given a graph G, a list assignment L and a *majority tolerance* $\alpha \in (0, 1)$, an α-*majority* L-*colouring* of G is a colouring $\omega : E \to C$ from the given lists such that for every $v \in V$ and each $c \in C$, the number of edges coloured c which are incident with v does not exceed $\alpha \cdot d(v)$. We discuss some restrictions necessary to extend this notion to a more general setting with diversified $\alpha = \alpha(c)$ majority tolerances for distinct colours $c \in C$. In particular, for any list assignment $L : E \to 2^C$ with $\sum_{c \in L(e)} \alpha(c) \geq 1 + \varepsilon$ and $|L(e)| \leq \ell$ for each edge e, we show that there exists an α-majority L-colouring of G, provided that $\delta(G) = \Omega(\ell^2 \varepsilon^{-2})$.

References

[1] F. Bock, R. Kalinowski, J. Pardey, M. Pilśniak, D. Rautenbach, M. Woźniak, Majority edge-colorings of graphs, Electron. J. Combin. 30 (2023), #P1.42.

[2] P. Pękała, J. Przybyło, On generalised majority edge-colourings of graphs, Electron. J. Combin. 31 (2024), #P.4.66.

[3] P. Pękała, J. Przybyło, On list extensions of the majority edge colourings, arXiv:2502.12688 (2025).

On a conjecture of Faudree and Schelp

Michiel Provoost

(joint work with Jan Goedgebeur, Jorik Jooken, and Carol T. Zamfirescu)

In 1976 Faudree and Schelp [2] conjectured that in a hamiltonian-connected graph G on n vertices for any two distinct vertices $x, y \in V(G)$, for any length k with $n/2 \leq k \leq n - 1$ there exists a path of length k between x and y. In 1978 Thomassen [1] discovered a (non-planar) family of counterexamples to this conjecture, showing that there exist hamiltonian-connected n-vertex graphs containing two vertices with no path of length $n - 2$ between them. His smallest counterexample has order 27.

Using computational methods we discovered a family of cubic planar counterexamples of $16+6k$ vertices for every $k \geq 0$ that are hamiltonian-connected and have a pair of vertices having no paths of lengths $\{3, 5, 7, \ldots, 9+4k\}$ between them. Thus we have found a family with a pair of vertices missing a linear number of path lengths (in the order of the graph) within the interval conjectured by Faudree and Schelp.

References

[1] C. Thomassen, Counterexamples to Faudree and Schelp's conjecture on hamiltonian-connected graphs, J. Graph Theory 2 (1978), 341–347.

[2] R.J. Faudree, R.H. Schelp, The square of a block is strongly path connected, J. Combin. Theory Ser. B 20 (1976), 47–61.

On colourings and Alon-Tarsi number of hypergraphs

Jakub Przybyło

(joint work with Marcin Anholcer, Bartłomiej Bosek, Grzegorz Gutowski, Michał Lasoń, Oriol Serra, Michał Tuczyński, Lluís Vena, and Mariusz Zając)

Given a hypergraph $H = (V, E)$, let us associate with each edge $e \in E$ a linear expression with non-zero coefficients whose variables correspond to the vertices of e. Let p_H denote the product of all such linear expressions over the edges of H. Our primary objective was to investigate the relationship between the Alon–Tarsi number of p_H, $\mathrm{AT}(p_H)$, and the edge density of H, $\mathrm{ed}(H)$.

We observed that $\mathrm{AT}(p_H) = \lceil \mathrm{ed}(H) \rceil + 1$ when all coefficients in p_H are equal to 1. Our main result implies that, regardless of the choice of coefficients, if these are not uniform in any edge, they can always be rearranged within the edges so that, for the resulting polynomial p'_H, the inequality

$$\mathrm{AT}(p'_H) \leq 2 \lceil \mathrm{ed}(H) \rceil + 1$$

holds. We moreover conjecture that such a rearrangement of coefficients is, in fact, unnecessary.

Both our results and our conjecture have several implications, particularly those related to list colourings of hypergraphs, but also a number of less obvious ones.

References

[1] M. Anholcer, B. Bosek, G. Gutowski, M. Lasoń, J. Przybyło, O. Serra, M. Tuczyński, L. Vena, M. Zając, Alon-Tarsi for hypergraphs, arXiv:2501.00157 (2025).

Generation of uniquely 3-edge-colourable graphs

Jozef Rajník

(joint work with Nadiya Balanchuk and Edita Máčajová)

A graph is *uniquely k-edge-colourable* if it admits only one k-edge-colouring up to a permutation of colours. For $k > 3$, each such graph is isomorphic to the star $K_{1,k}$. The missing characterisation of uniquely 3-edge-colourable (U3EC) graphs provides an interesting open problem that is also related to a smallest counterexample to the Cycle double cover conjecture. Except $K_{1,3}$, each U3EC graph needs to be cubic. Each known U3EC cubic graph can be obtained by a repeated 3-join starting from K_4 or the generalised Petersen graph $PG(9, 2)$ of order 18. The latter one is the only known example with cyclic connectivity 5, while all the other known examples have cyclic connectivity 4.

In this talk, we will question the existence of other U3EC cubic graphs. We will present two algorithms for the generation of U3EC cubic graphs, one general and one for graphs with cyclic edge-connectivity 4. Employing these algorithms, we verified that there is no U3EC graph with cyclic edge-connectivity 4 and order at most 28.

Cyclic edge-connectivity of junctions of multipoles

Erik Řehulka

(joint work with Jozef Rajník)

Cyclic edge-connectivity is an interesting property of cubic graphs, that has proven to be significant in graph theory. Research has shown, that high cyclic edge-connectivity correlates with certain structural properties of cubic graphs, and notably, smallest potential counterexamples to several open problems need to have high cyclic edge-connectivity. However, theoretical tools for determining cyclic edge-connectivity remain limited. In this work, we study cubic graphs built from smaller building blocks, for which we employ the concept of multipoles [1], and demonstrate how we can determine the cyclic edge- connectivity of the resulting graph, by imposing certain conditions on these blocks. Our developed tools provide a framework for proving that particular constructions achieve desired cyclic edge-connectivity.

References

[1] M.A. Fiol, A boolean algebra approach to the construction of snarks, in: Y. Alavi, G. Chartrand, O.R. Oellermann, A.J. Schwenk, Graph Theory, Combinatorics and Applications, Vol. 1, Wiley (1991), 493–524.

HIST-critical graphs
and Malkevitch's conjecture

Jarne Renders
(joint work with Jan Goedgebeur, Kenta Noguchi, and Carol T. Zamfirescu)

In a given graph, a *homeomorphically irreducible spanning tree (HIST)* is a spanning tree without 2-valent vertices. They are antithetical to hamiltonian cycles in the sense that they are connected spanning subgraphs in which no vertex has degree 2. Motivated by developing a better understanding of *HIST-free* graphs, i.e. graphs that do not contain a HIST and mirroring the study of hypohamiltonian graphs, we study *HIST-critical* graphs. These are HIST-free graphs in which every vertex-deleted subgraph does contain a HIST. Using a combination of theoretical and computer-based methods, we give an almost complete characterisation of the orders for which HIST-critical graphs exist and present infinite families of planar examples in which nearly all vertices are 4-valent. (See the left-hand side of Figure 2.)

Malkevitch [1] conjectured in 1979 that all 4-connected planar graphs have a HIST, mirroring Tutte's famous result on hamiltonian cycles. We computationally verify Malkevitch's conjecture up to order 22, enumerate the number of HISTs in antiprisms, a family of planar 4-connected graphs with few HISTs (see the right- hand side of Figure 2) and confirm Malkevitch's conjecture for the family of line graphs of cyclically 4-edge-connected cubic graphs.

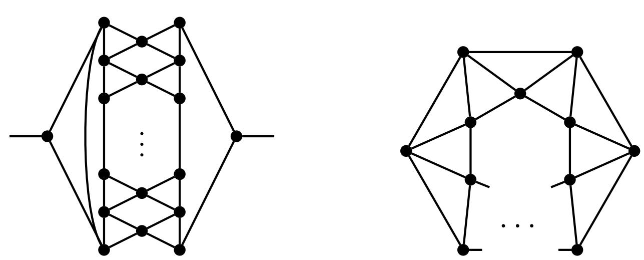


Figure 2: **Left-hand side:** A family of HIST-critical planar graphs with few cubic vertices. **Right-hand side:** The family of antiprism graphs.

References

[1] J. Malkevitch. Spanning trees in polytopal graphs. Ann. New York Acad. Sci. 319 (1979), 362–367.

Independent mutual-visibility coloring

Babak Samadi

(joint work with Boštjan Brešar, Iztok Peterin, and Ismael G. Yero)

Given a graph G, a subset $M \subseteq V(G)$ is a mutual-visibility (MV) set if for every $u, v \in M$, there exists a u, v-geodesic whose internal vertices are not in M ([2]). We investigate proper vertex colorings of graphs whose color classes are mutual-visibility sets. The main concepts that arise in this investigation are independent mutual-visibility (IMV) sets and vertex partitions into these sets (IMV colorings). The IMV number μ_i and the IMV chromatic number χ_{μ_i} are defined as maximum and minimum cardinality taken over all IMV sets and IMV colorings, respectively. Along the way, we also continue with the study of MV chromatic number χ_μ (as the smallest number of sets in a vertex partition into MV sets), which was initiated in [3].

We establish a close connection between the (I)MV chromatic numbers of subdivisions of complete graphs and Ramsey numbers $R(4^k; 2)$. From the computational point of view, we prove that the problems of computing χ_{μ_i} is NP-complete, and that it is NP-hard to decide whether a graph G satisfies $\mu_i(G) = \alpha(G)$, even for graphs with diameter 4, where $\alpha(G)$ is the independence number of G (this is the completion of the statements $\mu_i(G) \leq \alpha(G)$ for any graph G and $\mu_i(G) = \alpha(G)$ for any graph G with $\mathrm{diam}(G) \leq 3$ given in [1]).

Tight bounds on χ_{μ_i}, χ_μ and μ_i, as well as, exact values/formulas for these parameters in some classical families of graphs are given. In particular, we prove that $\chi_{\mu_i}(T) = \chi_\mu(T)$ holds for any tree T of order at least 3, and determine their exact formulas in the case of lexicographic product graphs. Finally, we give tight bounds on the (I)MV chromatic numbers for the Cartesian product graphs, which lead to exact values in some important families of product graphs.

References

[1] S. Cicerone, G. Di Stefano, S. Klavžar, On the mutual-visibility in Cartesian products and in triangle-free graphs, Appl. Math. Comput. 438 (2023), 127619.

[2] G. Di Stefano, Mutual visibility in graphs, Appl. Math. Comput. 419 (2022), 126850.

[3] S. Klavžar, D. Kuziak, J.C. Valenzuela-Tripodoro, I.G. Yero, Coloring the vertices of a graph with mutual-visibility property, arXiv:2408.03132 (2024).

Random embeddings of graphs

Robert Šámal

(joint work with Jesse Campion Loth, Babak Ghanbari, Kevin Halasz, Radek Hušek, Tomáš Masařík, Bojan Mohar, and Matěj Prokopič)

A combinatorial embedding of a graph G is given as a tuple (π, λ), where $\pi = (\pi_v)_{v \in V(G)}$ is a collection of *local rotations*: π_v is a cyclic ordering (permutation with one cycle) of edges incident with v. Further, $\lambda = (\lambda_e)_{e \in E(G)}$ is a collection of signs on edges: $\lambda_e = 1$ is a typical "planar" edge, while $\lambda_e = -1$ is an edge going through a crosscap, that makes a "twist" between left and right side of the face. See [4] for details.

A random embedding, introduced by Stahl [5], is given by choosing uniformly at random π_v from all unicyclic permutations, with all choices independent. We let $\lambda_e = 1$ for all edges in the orientable case; in an *nonorientable random embedding* we add to this a random choice of $\lambda_e \in \{\pm 1\}$ for each $e \in E(G)$.

We will discuss results about random graph embeddings obtained in the past years by the speaker and his collaborators [1, 2, 3, 6]. In particular, we will provide bounds on the expected number of faces and estimates of the probability that an edge is singular (a loop of the dual).

References

[1] J. Campion Loth, K. Halasz, T. Masařík, B. Mohar, and R. Šámal, Random 2-cell embeddings of multistars, Proc. Amer. Math. Soc. 150 (2022), 3699–3713.

[2] J. Campion Loth, B. Mohar, Expected number of faces in a random embedding of any graph is at most linear, Combin. Probab. Comput. 32 (2023), 682–690.

[3] J. Campion Loth, K. Halasz, T. Masařík, B. Mohar, R. Šámal, Random embeddings of graphs: the expected number of faces in most graphs is logarithmic, Proc. SODA (2024), 1177–1193.

[4] B. Mohar, C. Thomassen, Graphs on surfaces, Johns Hopkins University Press, 2001.

[5] S. Stahl, Permutation-partition pairs, A combinatorial generalization of graph embeddings, Trans. Amer. Math. Soc. 259 (1980), 129–145.

[6] R. Šámal, Loops in random duals, EuroComb 2025.

3-colourability, diamonds and butterflies

Ingo Schiermeyer

(joint work with Nadzieja Hodur, Monika Pilśniak, and Magdalena Prorok)

The 3-colourability problem is an NP-complete problem which remains NP-complete for graphs with maximum degree four, for claw-free graphs, and even for (claw, diamond)-free graphs. In this talk we will consider induced subgraphs, among them are the *claw* ($K_{1,3}$), the *diamond* (the graph $K_4 - e$), the *butterfly* (two triangles sharing a vertex), and the generalized *net* $N_{i,j,k}$ (a triangle with three attached paths with i, j, k edges).

Our main result is a complete characterization of all 3-colourable (*claw*, *diamond*, H)-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$. We will present a description of all non 3-colourable (*claw*, *diamond*, H)-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$ in terms of butterflies. Moreover, we will show extensions of this characterization to larger graph classes.

References

[1] B. Randerath, I. Schiermeyer, Vertex colouring and forbidden subgraphs — a survey, Graphs Combin. 20 (2004), 1–40.

[2] I. Schiermeyer, B. Randerath, Polynomial χ-binding functions and forbidden induced subgraphs: A survey, Graphs Combin. 35 (2019), 1–31.

Are there any permutation snarks of order 6 (mod 8)?

Martin Škoviera

(joint work with Edita Máčajová)

A permutation snark is a cubic graph that has a 2-factor consisting of two chordless cycles and is not 3-edge-colourable. The order of every permutation snark is twice an odd number and thus an integer congruent to either 2 or 6 (mod 8). While there exist infinitely many permutation snarks whose orders cover all the integers congruent to 2 (mod 8), no permutation snarks of order 6 (mod 8) are known. Exhaustive computer searches (Brinkmann et al. [1] and Goedgebeur and Renders [2]) have revealed that if such a permutation snark exists, then its order must be at least 54. Brinkmann et al. [1] also asked whether all permutation snarks have order 2 (mod 8). Several researchers have attempted answering this question, with no success. In this talk we link the existence of permutation snarks of order 6 (mod 8) to the existence of non-critical permutations snarks, which are also unknown. Our main result states that if there exists a non-critical permutation snark, then there is also one on 6 (mod 8) vertices.

References

[1] G. Brinkmann, J. Goedgebeur, J. Hägglund, K. Markström, Generation and properties of snarks, J. Combin. Theory Ser. B 103 (2013), 468–488.

[2] J. Goedgebeur, J. Renders, Generation of cycle permutation graphs and permutation snarks, in: R. Královič, V. Kůrková (eds.), Proc. SOFSEM 2025, LNCS 15538 (2025), pp. 333–346.

Defective vertex coloring of toroidal graphs

Diana Švecová

(joint work with Mária Maceková and Roman Soták)

Improper vertex coloring is such an assignment of colors to the vertices of a graph where adjacent vertices do not necessarily receive distinct colors. We say a graph G is $(d_1, \ldots, d_k)$-colorable if its vertices can be partitioned into k sets $V_1, \ldots, V_k$ such that $\Delta(\langle V_i \rangle_G) \leq d_i, i \in \{1, \ldots, k\}$. We call this coloring defective, the number d_i is called the defect of the color i.

While there are many results about such colorings for the class of planar graphs, not much is known about improper colorings of graphs embeddable on other surfaces. We focus on defective coloring of graphs embeddable on torus. In the talk, we will give a brief overview of known results and present the contribution our research has brought. Specifically, we will give tight upper bounds for color defects when using k colors, $k \in \{4, 5, 6\}$. We will sketch the proof demonstrating that toroidal graphs admit a $(0, 0, 0, 3)$-coloring.

Nordhaus–Gaddum-type theorems for many subgraphs: Maximum average degree and more

Zsolt Tuza

(joint work with Yair Caro)

The classical theorem of Nordhaus and Gaddum [1] states that if G is a graph of order n, then the chromatic number of G and of its complement $\overline{G}$ satisfy the inequalities $2\sqrt{n} \leq \chi(G)+\chi(\overline{G}) \leq n+1$ and $n \leq \chi(G)\cdot\chi(\overline{G}) \leq (n+1)^2/4$. Let now k be any integer in the range $2 \leq k \leq \binom{n}{2}$. We consider edge decompositions of the complete graph K_n into k mutually edge-disjoint subgraphs $G_1, \ldots, G_k$; i.e., $E(G_1) \cup \cdots \cup E(G_k) = E(K_n)$. Denoting by $\mathrm{Mad}(G) := \max\{2|E(H)|/|V(H)| : H \subseteq G\}$ the maximum average degree of graph G, we study the behavior of

$$M(k,n) := \max_{G_1,\ldots,G_k} \sum_{i=1}^{k} \mathrm{Mad}(G_i)\,.$$

We prove a general formula for $M(k, n)$ which is asymptotically tight as k grows, and it is even exact in many cases. The obtained bounds provide strong estimates for the analogous problems on clique number, chromatic number, list coloring (choice) number, and Szekeres–Wilf (coloring) number, as well as on maximum local vertex- and edge-connectivity. For instance, already for not too small k, the exponential error term in the known bound $n + 7^k$ concerning chromatic number [2] can be replaced with a formula which is asymptotically optimal.

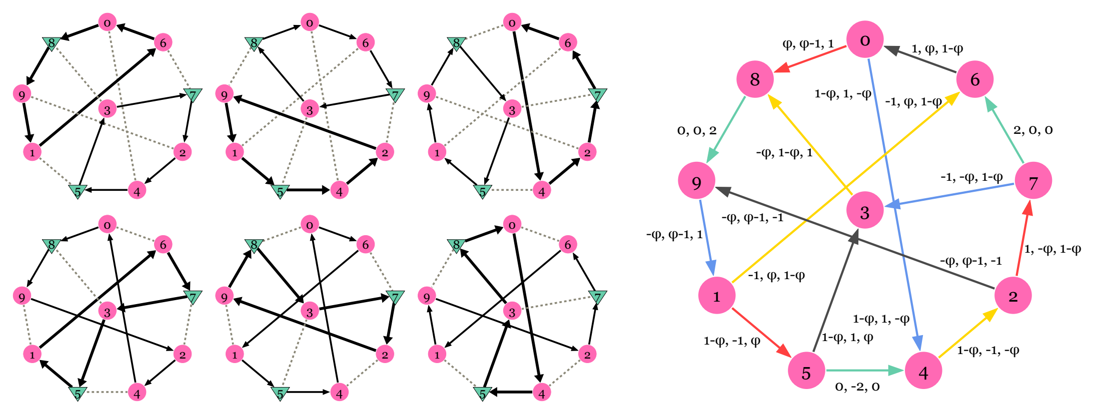
References

[1] E.A. Nordhaus, J. Gaddum, On complementary graphs, Amer. Math. Monthly 63 (1956), 175–177.

[2] Z. Füredi, A.V. Kostochka, M. Stiebitz, R. Škrekovski, D.B. West, Nordhaus–Gaddum-type theorems for decompositions into many parts, J. Graph Theory 50 (2005), 273–292.

Oriented Berge–Fulkerson conjecture and unit vector flows on S^2

Nikolay Ulyanov



We study 2 constructions from the world of cubic bridgeless graphs.

The Berge–Fulkerson conjecture [1] states that every bridgeless cubic graph can be covered with six perfect matchings such that each edge is covered exactly twice. An equivalent reformulation is that it's possible to find six 2-factors that cover each edge 4 times. Our main contribution is the introduction of the oriented version of this construction. We prove the existence of the construction for Isaacs flower snarks. We also construct an orientable surface, with faces corresponding to cycles of the 4-cover. In the case when the surface has no boundary, we prove that we can also build an orientable cycle double cover (with 6 cycles). In another case, when we can build a nowhere-zero 5-flow by summing cycles with weights $\left(0, 0, \frac{1}{3}, \frac{2}{3}, -\frac{4}{3}, -\frac{8}{3}\right)$, we find a cycle double cover with 5 cycles.

We also study two unit vector flows conjectures, proposed by K. Jain [2]. Second conjecture asserts an existence of a nowhere-zero 5-flow on a whole unit sphere S^2, for which we find several counterexamples. The first conjecture asserts that we can map edges of bridgeless graphs into the unit sphere S^2. Here we build various nowhere-zero 5-flow subsets of S^2 suitable for some of the snarks. Finally, we show that a specific S^2 subset is related to a an almost strong Petersen colouring, which is a relaxed version of the strong Petersen colouring, which empirically exists for a large proportion of snarks, and implies both oriented Berge–Fulkerson and oriented 5-cycle double cover conjectures.

References

[1] D.R. Fulkerson, Blocking and anti-blocking pairs of polyhedra, Math. Program. 1 (1971), 168–194.

[2] K. Jain, Open Problem Garden, Unit vector flows (http://www.openproblemgarden.org/op/unit_vector_flows).

Computational methods for finding bi-regular cages

Tibo Van den Eede

(joint work with Jan Goedgebeur and Jorik Jooken)

The cage problem is a renowned problem in extremal graph theory, asking for (r, g)-*graphs* of minimum order. An (r, g)-graph is an r-regular graph of girth g. An (r, g)-graph of minimum order is called an (r, g)-*cage*, or simply a *cage*, and its order is denoted by $n(r, g)$. Figure 3 illustrates the only $(3, 6)$-cage, also known as the Heawood graph. Since this cage has 14 vertices, $n(3, 6)$ is equal to 14.

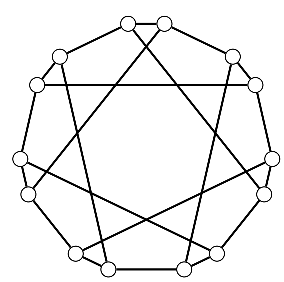


Figure 3: The Heawood graph.

It is known that (r, g)-graphs exist for every integer $r \geq 2$ and $g \geq 3$ [1], but the exact values of $n(r, g)$, and the corresponding cages, have only been determined for a limited set of (r, g) pairs. For the remaining cases, there are lower and upper bounds that have been progressively tightened over the years.

Due to the difficulty of the problem, researchers started to investigate variants of (r, g)-cages in the hope of gaining new insights. One such variant is the $(\{r, m\}; g)$-*cage*, also denoted as the *bi-regular cage*. An $(\{r, m\}; g)$-cage is a graph of minimum order with girth g and vertices of degree r and m, where $r < m$.

Although computer-assisted methods have driven significant progress for regular cages, they have been far less exploited in the bi-regular setting. In this talk, we address this gap by presenting three computational methods. As a result, we produce, previously unknown, exhaustive lists of bi-regular cages for 24 triples (r, m, g) and improve lower and upper bounds for 122 triples.

References

[1] H. Sachs, Regular graphs with given girth and restricted circuits, J. London Math. Soc. 38 (1963), 423–429.

Participants

Barát János
Alfréd Rényi Institute of Mathematics, Budapest, Hungary
University of Pannonia, Veszprém, Hungary

Kenny Bešter Štorgel
Faculty of Information Studies, Novo mesto, Slovenia
University of Primorska, Koper, Slovenia

Blázsik Zoltán L.
University of Szeged, Szeged, Hungary

Bujtás Csilla
University of Ljubljana, Ljubljana, Slovenia
Institute of Mathematics, Physics, and Mechanics, Ljubljana, Slovenia

Choi Ilkyoo
Hankuk University of Foreign Studies, Yongin-si, Republic of Korea
Institute for Basic Science, Daejeon, Republic of Korea

Cichacz-Przeniosło Sylwia
AGH University, Kraków, Poland

Detlaff Magda
University of Gdańsk, Gdańsk, Poland

Dvořák Zdeněk
Charles University, Prague, Czech Republic

Fabrici Igor
P.J. Šafárik University, Košice, Slovakia

Furmańczyk Hanna
University of Gdańsk, Gdańsk, Poland

Grzelec Igor
AGH University, Kraków, Poland

Haxell Penny E.
University of Waterloo, Waterloo, Canada

Hedžet Jaka
Institute of Mathematics, Physics, and Mechanics, Ljubljana, Slovenia

Horňák Mirko
P.J. Šafárik University, Košice, Slovakia

Jendrol' Stanislav
P.J. Šafárik University, Košice, Slovakia

Kalinowski, Rafał
AGH University, Kraków, Poland

Kamyczura Mateusz
AGH University, Kraków, Poland

Kardoš František
LaBRI, Université de Bordeaux, Bordeaux, France

Keszegh Balázs
Alfréd Rényi Institute of Mathematics, Budapest, Hungary
Eötvös Loránd University, Budapest, Hungary

Knor Martin
Slovak University of Technology, Bratislava, Slovakia

Lužar Borut
Faculty of Information Studies, Novo mesto, Slovenia

Máčajová Edita
Comenius University, Bratislava, Slovakia

Maceková Mária
prev. P.J. Šafárik University, Košice, Slovakia

Madaras Tomáš
P.J. Šafárik University, Košice, Slovakia

Masařík Tomáš
University of Warsaw, Warsaw, Poland

Masaříková Jana
University of Warsaw, Warsaw, Poland

Matok Matúš
Comenius University, Bratislava, Slovakia

Mattiolo Davide
Katholieke Universiteit Leuven, Kortrijk, Belgium

Mazák Ján
Comenius University, Bratislava, Slovakia

Mikšaník David
Charles University, Prague, Czech Republic

Mütze Torsten
University of Kassel, Kassel, Germany

Oguagbaka Makuochukwu Felix
Comenius University, Bratislava, Slovakia

Onderko Alfréd
P.J. Šafárik University, Košice, Slovakia

Pękała Paweł
AGH University, Kraków, Poland

Provoost Michiel
Katholieke Universiteit Leuven, Kortrijk, Belgium

Przybyło Jakub
AGH University, Kraków, Poland

Rajník Jozef
Comenius University, Bratislava, Slovakia

Renders Jarne
Katholieke Universiteit Leuven, Kortrijk, Belgium

Řehulka Erik
Comenius University, Bratislava, Slovakia

Samadi Babak
Institute of Mathematics, Physics, and Mechanics, Ljubljana, Slovenia

Šámal Robert
Charles University, Prague, Czech Republic

Schiermeyer Ingo
AGH University, Kraków, Poland
Technische Universität Bergakademie, Freiberg, Germany

Škoviera Martin
Comenius University, Bratislava, Slovakia

Soták Roman
P.J. Šafárik University, Košice, Slovakia

Švecová Diana
P.J. Šafárik University, Košice, Slovakia

Tuczyński Michał
Warsaw University of Technology, Warsaw, Poland

Tuza Zsolt
Alfréd Rényi Institute of Mathematics, Budapest, Hungary
University of Pannonia, Veszprém, Hungary

Ulyanov Nikolay
Independent researcher, Berlin, Germany

Van den Eede Tibo
Katholieke Universiteit Leuven, Kortrijk, Belgium